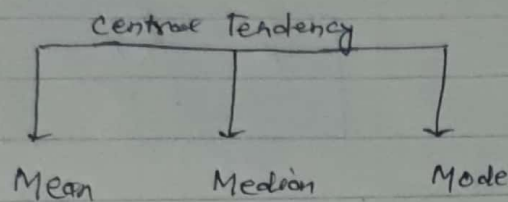


Central Tendency and Dispersion:

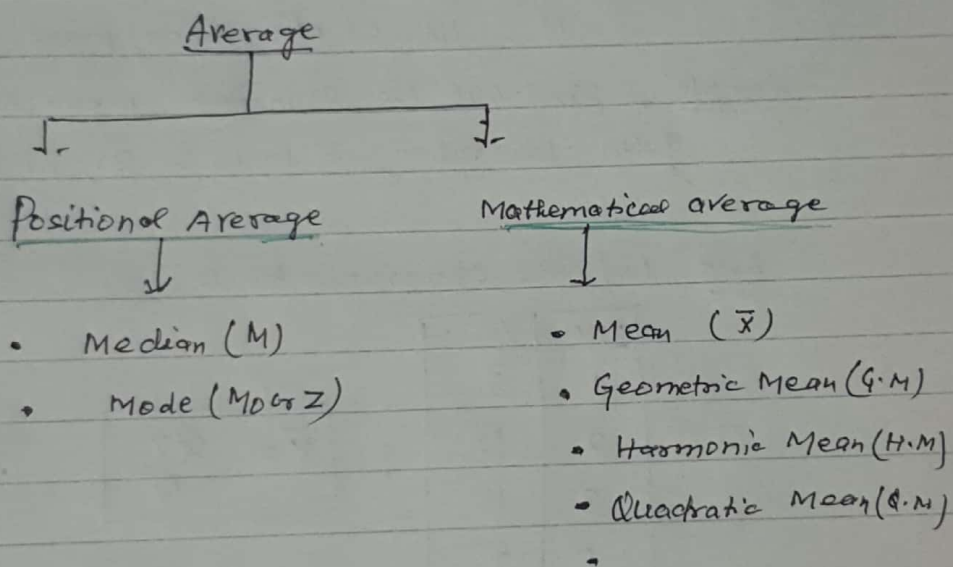
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Central Tendency \rightarrow Measures of central tendency or average reduce the large number of observations to one figure. Actually the measures of central tendency describe the tendency of items of a group around the middle in frequency distribution of numerical value.

There are three types of central tendency -



But, statistical average are two types



Mean (Arithmetic Mean): Arithmetic Mean is the most widely used method of calculating average, so much so that when only mean is indicated it is assumed to be arithmetic mean universally. It is obtained by adding up all the observations and dividing it by the number of observations.

Arithmetic Mean for Individual Series Data:-

For Individual series data arithmetic mean may be

calculated by applying any of the following methods.

i) Direct Method (प्रत्यक्ष विधि)

ii) Short cut Method (अल्प विधि)

Direct Method

It is calculated by the following formula.

$$\bar{X} = \frac{\sum X}{N}$$

Where,

\bar{X} = Arithmetic Mean - (समांतर माध्य)

$\sum X$ = Sum of observation/Items (पदों का योग)

N = No. of observation/Items (पदों की संख्या)

Example - Find out the arithmetic mean of the following given observation, 5, 10, 15, 20, 25, 30, 35,

Soln Let the observation be x

S.No	X
1	5
2	10
3	15
4	20
5	25
6	30
7	35
$N = 7$	140

$$\bar{X} = \frac{\sum x}{N}$$

$$= \frac{140}{7}$$

$$\bar{X} = 20 \text{ Ans.}$$

Short cut Method :- It is calculated by the following formula

$$\bar{X} = A + \frac{\sum dx}{N}$$

where $A =$ Assumed Mean (क्याहीया मीका)

$d =$ Deviation from assumed mean ($d = x - A$)

$N =$ NO. of observation/items.

Example :- Find out the arithmetic mean of the following given data observations :- 10, 15, 20, 25, 30, 35, 40

Soln :- let the observation be x

S/NO	x	$dx = (x - A)$
1	10	-15
2	15	-10
3	20	-5
4	<u>25(A)</u>	0
5	30	+5
6	35	+10
7	40	+15
$N = 7$		$\sum dx = 0$

Assumed Mean = 25

$$\bar{x} = A + \frac{\sum dx}{N}$$

$$= 25 + \frac{0}{7}$$

$$\bar{x} = 25 \text{ Ans}$$

Arithmetic Mean for discrete series :-

For discrete series arithmetic mean may be calculated by applying the following ~~formula~~ methods

- i) Direct Method - (समझ विधि)
- ii) Short cut Method (कट्टु विधि)
- iii) step deviation Method (सुग विधि विधि)

Example :- Calculate Arithmetic mean from the following data

Marks :- 10, 20, 30, 40, 50, 60, 70, 80

No. of students :- 2, 5, 9, 10, 12, 7, 3, 2

Soln:

Marks	No of students	fx
x	f	
10	2	20
20	5	100
30	9	270
40	10	400
50	12	600
60	7	420
70	3	210
80	2	160
	$N = 50$	$\Sigma fx = 2180$

Direct Method

$$\bar{x} = \frac{\Sigma fx}{N}$$

where,

\bar{x} = Mean.

N = Sum of frequency

Σfx = Product of x & f

$$\bar{x} = \frac{\Sigma fx}{N} = \frac{2180}{50} = \frac{218}{5} = 43.6$$

$$\bar{x} = 43.6 \text{ Ans.}$$

(ii) Short Cut Method

~~$$\bar{x} = \frac{\Sigma fx}{N}$$~~

$$\bar{x} = A + \frac{\Sigma fdx}{N}$$

Example! - Calculate arithmetic mean by short cut method from the data given

Marks(x)	No. of student(f)	dx	$f dx$
10	2	-30	-60
20	5	-20	-100
30	9	-10	-90
40	10	0	0
50	12	+10	+120
60	7	+20	+140
70	3	+30	+90
80	2	+40	+80
	$N = 50$		$\Sigma f dx = 180$

$$\bar{x} = A + \frac{\sum f dx}{N}$$

$$= 40 + \frac{189}{50} = 40 + 3.6$$

$$\bar{x} = 43.6 \quad \text{Ans.}$$

(iii) Step deviation Method: - (यदि विचलन विपर) - It is calculated by the following formula:-

$$\bar{x} = A + \frac{\sum f dx}{N} \times i$$

Where, dx = Deviation from assumed mean by step deviation

i = Difference of size of items

N = $\sum f$ = Sum of frequency

A = Assumed mean.

Example: - Calculate arithmetic mean by step deviation method from the data given in above example.

Marks x	No of students f	Deviation from Assumed mean dx	f dx
10	2	-4	-8
20	5	-3	-15
30	9	-2	-18
40	10	-1	-10
50	12	0	0
60	7	+1	+7
70	3	+2	+6
80	2	+3	+6
	N = 50		$\sum f dx$ = -32

$$\begin{aligned} \bar{X} &= A + \frac{\sum fdx}{N} \times i \\ &= 50 + \frac{-32}{50} \times 10 \\ &= 50 + \frac{-320}{50} \\ &= 50 - \frac{320}{50} \\ &= 50 - 6.4 = 43.6 \end{aligned}$$

$\bar{X} = 43.6$ Ans.

Arithmetic Mean for Continuous Series:-

The continuous series express the data, which is very vast. The calculation of arithmetic mean of this series is similar to that of discrete series after calculating the mid point or mid value of each segment of the continuous series which is called the class interval. The continuous series may have three types of class interval:-

- i) exclusive class interval, e.g. - 0-10, 10-20, 20-30
 - ii) Inclusive class interval e.g. - 0-9, 10-19
- If the data is given in the form of inclusive class interval, it is first converted into exclusive class interval
- iii) Commulative class interval, for example, more than 10, more than 20 etc or less than 10, less than 20 - etc

Following methods are applying for the calculation of continuous series.

- i) Direct Method
- ii) short cut Method
- iii) step deviation method

1) Direct Method

$$\text{Mid value (M)} = \frac{\text{Lower limit} + \text{Upper limit}}{2}$$

$$\bar{X} = \frac{\sum fM}{N}$$

where, m = mid value / mid point / central value
 f = frequency
 $N = \sum f$ = sum of frequency.

Example - From the following data calculate arithmetic average by direct method

Marks :- 0-10, 10-20, 20-30, 30-40, 40-50, 50-60, 60-70, 70-80
NO of students :- 2, 7, 10, 15, 20, 16, 6, 4

Soln.

Marks C.L	NO. of students (f)	Mid value (M)	fM
0-10	2	5	10
10-20	7	15	105
20-30	10	25	250
30-40	15	35	525
40-50	20	45	900
50-60	16	55	880
60-70	6	65	390
70-80	4	75	300
	N = 80		$\sum fM$ = 3360

$$\bar{X} = \frac{\sum fM}{N} = \frac{3360}{80}$$
$$= \frac{336}{8} = 42$$

$$\bar{X} = 42 \text{ Ans}$$

(ii) Short cut Method

$$\bar{x} = A + \frac{\sum f dx}{N}$$

where,

\bar{x} = Arithmetic Mean

A = Assumed Mean

f = Frequency.

dx = Deviation from assumed mean.

N = sum of frequency.

Example :- Calculation arithmetic average by short cut method from the data given in above example.

Marks (C.I.)	No. of students (f)	Mid value (M)	Deviation from 45 (dx)	f dx
0-10	2	5	-40	-80
10-20	7	15	-30	-210
20-30	10	25	-20	-200
30-40	15	35	-10	-150
40-50	20	45	0	0
50-60	16	55	+10	+160
60-70	6	65	+20	+120
70-80	4	75	+30	+120
	N = 80			$\sum f dx = -240$

$$\bar{x} = A + \frac{\sum f dx}{N}$$

$$= 45 + \frac{-240}{80}$$

$$= 45 - \frac{240}{80}$$

$$= 45 - 3$$

$$\bar{x} = 42 \text{ Ans.}$$

(iii) Step Deviation Method

$$\bar{X} = A + \frac{\sum f dx'}{N} \times i$$

Where,

- \bar{X} = Arithmetic Mean
- A = Assumed Mean
- dx = Deviation from assumed mean
- N = sum of frequency
- f = Frequency
- i = Magnitude/range (Interval)

Example Calculate arithmetic mean by step deviation method from the above example given.

Soln.

Marks	No. of students	Mid value	Deviation from	dx'	f dx'
Cl.	(f)	(M)	35 (dx)		
0-10	2	5	-30	-3	-6
10-20	7	15	-20	-2	-14
20-30	10	25	-10	-1	-10
30-40	15	35	0	0	0
40-50	20	45	+10	+1	+20
50-60	16	55	+20	+2	+32
60-70	6	65	+30	+3	+18
70-80	4	75	+40	+4	+16
	N = 80				$\sum f dx' = 56$

$$\begin{aligned} \bar{X} &= A + \frac{\sum f dx'}{N} \times i \\ &= 35 + \frac{56}{80} \times 10 \\ &= 35 + 7 = 42 \\ \boxed{\bar{X} = 42} \text{ Ans.} \end{aligned}$$

Example: - Calculate mean from the following data: -

Wage in Rs 1-5, 6-10, 11-15, 16-20, 21-25,
 No. of workers 5 7 10 6 2

<u>Soln.</u>	Wage in (Rs)	No. of workers	M.V	dx	dx'	$f dx'$
	C.L	f	x	(13)		
	1-5	5	3	-10	-2	-10
	6-10	7	8	-5	-1	-7
	11-15	10	13	0	0	0
	16-20	6	18	+5	+1	+6
	21-25	2	23	+10	+2	+4
		$N=30$				$\sum f dx' = -7$

$$\bar{x} = A + \frac{\sum f dx'}{N} \times i$$

$$= 13 + \frac{-7}{30} \times 5$$

$$= 13 + \frac{-35}{30}$$

$$= 13 - \frac{35}{30}$$

$$= 13 - 1.167$$

$$\boxed{\bar{x} = 11.833} \text{ Ans}$$

Calculation of arithmetic average when cumulative frequency is given.

Example: - Calculate arithmetic average with the help of following data: -

Marks less than. 10, 20, 30, 40, 50, 60
 No. of students 3 10 20 25 28 30

Soln

Marks	Frequency	MV	$f_2(25)$	dx'	$f dx'$
C.L.	f	x			
0-10	3	5	-20	-2	-6
10-20	10-3=7	15	-10	-1	-7
20-30	20-10=10	25	0	0	0
30-40	25-20=5	35	+10	+1	+5
40-50	28-25=3	45	+20	+2	+6
50-60	30-28=2	55	+30	+3	+6
	$N=30$				$\sum f dx' = +4$

$$\bar{x} = A + \frac{\sum f dx'}{N} \times i$$

$$= 25 + \frac{4}{30} \times 10$$

$$= 25 + \frac{40}{30}$$

$$= 25 + 1.33$$

$$\bar{x} = 26.33 \quad \text{Ans.}$$

Example Calculate arithmetic mean with the help of following data:

Wage in (Rs): - More than 0, 10, 20, 30, 40, 50

No. of labour: - 50, 46, 40, 20, 10, 3

Wage	MV(x)	f	f_2	$dx(35)$	$f dx$
0-10	5	50-46=4	20	-30	-120
10-20	15	46-40=6	90	-20	-120
20-30	25	40-20=20	500	-10	-200
30-40	35	20-10=10	350	0	0
40-50	45	10-3=7	315	+10	+70
50-60	55	3	165	+20	+60
			$\sum f_2 = 1440$		$\sum f dx = -310$

Direct Method

$$\bar{x} = \frac{\sum fx}{N}$$

$$= \frac{1440}{50}$$

$$\bar{x} = 28.8 \text{ Am}$$

Shortcut Method

$$\bar{x} = a + \frac{\sum fdx}{N}$$

$$= 35 + \frac{-310}{50}$$

$$= 35 - \frac{310}{50}$$

$$= 35 - 6.2 = 28.8$$

$$\bar{x} = 28.8 \text{ Am}$$

Calculation of Missing frequency

Calculate the missing frequency from the following data when the mean is 33

x:	5	15	25	35	45	55
f:	5	10	25	30	?	10

S/n. let missing frequency be x

x	f	fx
5	5	25
15	10	150
25	25	625
35	30	1050
45	x	45x
55	10	550
	N	$\sum fx$
	80+x	2400+45x

$$\bar{x} = \frac{\sum fx}{N}$$

$$33 = \frac{2400 + 45x}{80 + x}$$

$$33(80 + x) = 2400 + 45x$$

$$= 2640 + 33x = 2400 + 45x$$

$$= 33x - 45x = 2400 - 2640$$

$$\Rightarrow -12x = -240$$

$$\therefore x = \frac{-240}{-12}$$

$$\text{Missing } f = \boxed{x = 20}$$

Merits of A.M are :-

- Simple to understand
- Easy to compute
- Capable of further mathematical treatment
- Calculated on the basis of all the items of the series
- Can be calculated even if some values of the series are missing.
- It is least affected by fluctuations in sample.

Demerits of A.M are :-

- Extreme items have ~~higher~~ disproportionate effect, for example

$$\frac{50+10+10+10+10}{5} = \frac{90}{5} = 45$$

Whereas in reality 4 out of 5 students failed, therefore, 45 marks can't be termed as representative.

- When data is vast, the calculation become tedious.
- In case of open end classes, mean can only be calculated by making some assumptions.
- A.M. is not representative if series is asymmetrical.

