PROPERTIES OF ORDINARY LEAST SQUARE

6th sem
Econometrics

Dr. N. N. Malik
Department of Eco.
Visthapit College,

Ordinary Least Square:— In statistics, Ordinary least square (OLS) is type of linear least square method for estimating the unknown parameter in a linear regression model. (OLS) chooses the parameters of a linear function of a set of explanatory variables by the principle of Least squares minimizing the sum of the square of the difference between the observed dependent variable (value of the variable ~~beig~~ being observed) in the given dataset and those predicted by the linear function

PROPERTIES —

There are following properties in ordinary least square:—

1) **Linearity:—** The linear property of (OLS) estimator means that OLS belongs to the class of estimator that which are linear in Y, the dependent variable. OLS estimators are linear only with respect to the dependent variable and not neccessorily with respect to the independent variables.

In mathematically:— An estimator is linearity if its functions of the sample observation on Y.

$$\hat{\beta} = f(Y)$$

$$\hat{\alpha} = f(Y)$$

$$\hat{\beta} = \frac{\Sigma xy}{\Sigma x^2} \quad \text{(From deviation)}$$

$$\left[\because x = X - \bar{X}, \quad y = Y - \bar{Y} \quad \bar{X} \text{ and } \bar{Y} \text{ are constant} \right]$$

$$\frac{\Sigma x(Y - \bar{Y})}{\Sigma x^2}$$

$$= \frac{\Sigma xY - \bar{Y}\Sigma x}{\Sigma x^2}$$

$$= \frac{\sum xy - \bar{y} \times 0}{\sum x^2} \quad \left[\because \sum x = 0 \right]$$

$$\hat{\beta} = \frac{\sum xy}{\sum x^2}$$

$$\frac{x}{\sum x^2} = \sum ky$$

$$\hat{\beta} = \sum ky$$

$$\hat{\beta} = f(Y)$$

Now, $\boxed{\hat{\alpha} = f(Y)}$

$$\hat{\alpha} = \left(\sum \frac{1}{n} \cdot k\bar{y} \right) Y$$

$$\hat{\alpha} = f(y)$$

(2) Unbiasedness :- Unbiasedness is one of the most desirable properties of any estimator. The estimator should ideally be an unbiased estimator of true parameter/popn. value.

In mathematically the bias of an estimator is define as ~~at~~ the difference between its expected value and actual value.

$$\boxed{\text{Bias} = E(\hat{\beta}) - \beta}$$

An estimator is unbiased if Bias $= 0$

If,

$$E(\hat{\beta}) = \beta$$

$$E(\hat{\beta}) = \beta$$

$$E(\hat{\alpha}) = \alpha$$

Now,

$$\hat{\beta} = \sum ky \quad (\text{By linearity})$$

$$= \sum k(\alpha + \beta X + U)$$

$$= \sum k\alpha + \beta \sum kX + \sum kU$$

Taking expectation on both side.

$$E(\hat{\alpha}) = \alpha + E\left(\left(\sum \frac{1}{n} - k\bar{x}\right)U\right)$$

$$= \alpha + \sum\left(\frac{1}{n} - k\bar{x}\right) E(U)$$

$$= \alpha + \sum\left(\frac{1}{n} - k\bar{x}\right) \times 0 \quad [E(U) = 0]$$

$$\alpha + 0$$

$$E(\hat{\alpha}) = \alpha \quad \text{Varified.}$$

(iii) Best estimator (Minimum variance):-
If the estimator is unbiased but doesnt have the least variance - it is not the best. and- if the estimator has the least variance but is baised - it is not best. But if the estimator is both unbaised and has the least variance it is the best estimator.

So, this ~~estimator~~ property is the best estimator as compared with any other estimator of the economy. Prove this model use take the:-

$$y = \alpha + \beta x + U$$

$$V(\hat{\beta}) < V(\tilde{\beta})$$

$$V(\hat{\alpha}) < V(\tilde{\alpha})$$

(iv) Consistency:- An estimator is said to be consistent if its value approaches the actual, true parameter (Popn.) value as the sample size increase. An estimator is consistent if it satisfies two conditions:-

(a) It is asymptotically unbaised.

(b) Its variance converges to 0 as the sample size increases.

$$= \alpha \times 0 + \beta \times 1 + \Sigma kU \quad \left[\begin{array}{l} \because \Sigma k = 0 \\ \Sigma kx = 1 \end{array}\right]$$

$$\hat{\beta} = \beta + \Sigma kU$$

Now taking expection on both side

$$E(\hat{\beta}) = E(\beta + \Sigma kU) \quad (E = \text{Mean})$$

$$\beta + \Sigma k E(U)$$

Again,

$$E(\hat{\alpha}) = \alpha.$$

$$\hat{\alpha} = \Sigma\left(\frac{1}{n} - k\bar{x}\right)y \quad \text{(By linearit}$$

$$= \Sigma\left(\frac{1}{n} - k\bar{x}\right) \times (\alpha + \beta x + U)$$

$$= \Sigma\left(\frac{1}{n}\alpha + \frac{\beta x}{n} + \frac{1}{n}U - \alpha k\bar{x} - \beta\bar{x}kx$$

$$= \frac{n}{n}\alpha + \frac{\beta\Sigma x}{n} + \frac{\Sigma U}{n} - \alpha\bar{x}\Sigma k - \beta$$

$$- \Sigma k\bar{x}U$$

$$= \alpha + \beta\bar{x} + \frac{\Sigma U}{n} - \alpha\bar{x} \times 0 - \beta\bar{x} \times 1 -$$

$$\left[\begin{array}{l} \because \frac{\Sigma x}{n} = \bar{x} \\ \Sigma k = 0 \\ \Sigma kx = 1 \end{array}\right.$$

$$= \left(\alpha + \beta\bar{x} + \frac{\Sigma U}{n} - \beta\bar{x} - k\bar{x}\Sigma U\right)$$

$$= \cancel{\alpha + \beta\bar{x} +}$$

$$= \left(\alpha + \frac{\Sigma U}{n} - \Sigma k\bar{x}U\right)$$

$$= \alpha + \Sigma U\left(\frac{1}{n} - k\bar{x}\right)$$

$$\hat{\alpha} = \alpha + \Sigma\left(\frac{1}{n} - k\bar{x}\right)U$$