Semester - VI

Mathematical Economics - II

Unit-1 : Basic Matrix Algebra

1.1 : Matrices : Types

1.2 : Addition, Subtraction and Multiplication of Matrices

1.3 : Order of matrices, Transpose of a Matrix, minor, Cofactors and Inverse of a Matrix

1.4 : Determinants : properties and value of a determina

1.5 : Rank of Matrix

1.6 : Solution of Equation by matrix Inversion

Matrix Algebra

(आव्यूह)

Matrix :- (Definition)

A matrix is defined as a rectangular array of elements arranged in rows and columns.

or

A matrix is defined as an array of numbers (or algebric symbols) set out in rows and columns.

पंक्ति और स्तम्भ में किसी सुनिश्चित क्रम से व्यवस्थित संख्यायें जो आयताकार व्यूह में लिखी हो, मैट्रिक्स या आव्यूह कहलाती है।

Explanation: Suppose there are 3 brothers : (i) Ajit (A) (ii) Bhanu (B) and Chander (C) in a family and that

A has a set of 3 pants, 3 shirts, 2 Bush-Shirts and 1 Tie.

B has a set of 5 Pants, 5 shirts, No Bush-shirts and 2 Ties.

C has a set of 6 pants, 8 shirts, 5 Bush-shirts and No Tie.

We can arrange this data in the following convenient System-

$$
\begin{array}{ccc}
\text{Brothers} & & \begin{array}{cccc} \text{Pants} & \text{Shirts} & \text{Bush shirts} & \text{Ties} \\ (P) & (S) & (B) & (T) \end{array} \\
\downarrow & & \\
A & \longrightarrow & \\
B & \longrightarrow & \left[\begin{array}{cccc} 3 & 3 & 2 & 1 \\ 5 & 5 & 0 & 2 \\ 6 & 8 & 5 & 0 \end{array}\right] \begin{array}{l} \text{– 1st Row} \\ \rightarrow \text{2nd Row} \\ \rightarrow \text{3rd Row} \end{array} \\
C & \longrightarrow & \\
 & & \begin{array}{cccc} \downarrow & \downarrow & \downarrow & \downarrow \\ \text{1st Col.} & \text{2nd col.} & \text{3rd col.} & \text{4th col.} \end{array}
\end{array}
$$

This system comprises 3 rows and 4 columns. Number written in such a particular form of rows and columns and enclosed by [ ] are called a matrix.

Its general form is: (सामान्य रूप से $m \times n$ क्रम का आव्यूह निम्न प्रकार से व्यक्त किया जाता है)

$$
A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}
$$

- This is a matrix of rows (m) and n columns. It is denoted (a_{ij}) and is read m by n.

Order of a matrix

If the matrix has a total number of elements equal to mn arranged in m rows and n columns, It is said to be of the order of m by n, which is often written as $m \times n$ matrix.

" The size of a matrix is called its 'order'. The order is specified as

(Number of Rows) × (Number of columns)

" यदि किसी आव्यूह में m पंक्तियाँ तथा n स्तंभ हो तो आव्यूह का क्रम $m \times n$ लिखा जाता है तथा (m by n) क्रम का आव्यूह कहलाता है "

$$\begin{bmatrix} 21 & 62 & 33 & 93 \\ 44 & 95 & 66 & 13 \\ 77 & 38 & 79 & 33 \end{bmatrix} \rightarrow$$ Thus the order of the matrix is 3×4.

Types of matrix (आव्यूह के प्रकार)

(i) शून्य आव्यूह (Null matrix)

A Null matrix is basically a matrix, whose all elements are zero. In a matrix basically there are two elements, first one is diagonal matrix and another one is non-diagonal elements. In Null matrix both diagonal and off-diagonal elements are zero. Null matrix is also called as zero matrix.

एक ऐसा आव्यूह जिसका प्रत्येक अवयव शून्य हो, शून्य आव्यूह कहलाती है।

$$O = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$ – This is Null matrix of 2×3 order.

(ii) वर्ग की आव्यूह (Square matrix)

A Square matrix has the number of columns equal to the number of rows. A matrix in which the number of rows is equal to the number of columns is said to be a square matrix. Thus an $m \times n$ matrix is said to be a square matrix if $m = n$ and is known as a matrix of order 'n'.

$$A=\begin{bmatrix} a_{11} & a_{12} & a_{1n} \\ a_{21} & a_{22} & a_{2n} \\ a_{n1} & a_{n2} & a_{nn} \end{bmatrix}$$ This is square matrix of order $n\times n$.

$$A=\begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$$ = This is square matrix of order 2×2.

(3) इकाई आव्यूह (Identity matrix)

If a square matrix has all elements 0 and each diagonal elements are non-zero, it is called identity and denoted by I.

यदि किसी वर्ग आव्यूह में मुख्य विकर्ण पर 1 हो और अवयव शून्य हो तो इस तरह के आव्यूह को इकाई आव्यूह कहते हैं। इसे 'I' के द्वारा चिन्हित किया जाता है।

$$I_2=\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad I_3=\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$I_n=\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & I_n \end{bmatrix}$$

(4) विकर्ण आव्यूह (Diagonal matrix)

A square matrix $B=[b_{ij}]_{m\times n}$ is said to be diagonal matrix if all its non-diagonal elements are zero, that is matrix $B=[b_{ij}]_{m\times n}$ is said to be a diagonal matrix if $b_{ij}=0$ when $i\neq j$

ऐसे आव्यूह जो वर्ग आव्यूह हो और जिनके मुख्य विकर्ण के अवयवों को छोड़कर शेष सभी अवयव शून्य हो विकर्ण आव्यूह कहलाती है। इसके अन्तर्गत मुख्य विकर्ण पर समान अवयव होना आवश्यक नहीं है।

$$B=\begin{bmatrix} 3 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$ - This is diagonal matrix of order 3×3 and diagonal elements are 3, 4, 1.

(5) Scalar matrix (अदिश आव्यूह)

A diagonal matrix is said to be a scalar matrix if all the elements in its principal diagonal are equal to some non-zero constant. A diagonal matrix is said to be a scalar matrix if its diagonal elements are equal that is a square matrix $B = [b_{ij}]_{m \times n}$ is said to be a scalar matrix if

$$b_{ij} = 0 \text{ when } i \neq j$$

$$b_{ij} = K, \text{ when } i = j, \text{ for some constant } K$$

एक ऐसी वर्ग आव्यूह जिसके मुख्य विकर्ण पर समान अवयव हो और अन्य जगह हो तो इसे आदिश आव्यूह कहते हैं। इस प्रकार अदिश आव्यूह, विकर्ण आव्यूह की एक विशेष दशा है।

$$B = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

(6) त्रिभुजीय आव्यूह (Triangular matrix)

A square matrix in which either all the entries above the principal diagonal or all the entries below the principal diagonal are zero.

त्रिभुजीय आव्यूह एक ऐसी वर्ग आव्यूह है जिसके मुख्य विकर्ण के ऊपर या नीचे के सभी अवयव शून्य हो।

$$|A| = \begin{bmatrix} 3 & 1 & 2 \\ 0 & 5 & 7 \\ 0 & 0 & 8 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 5 & 0 \\ 2 & 7 & 9 \end{bmatrix}$$

(7) Upper Triangular Matrix (ऊपर त्रिभुजीय आव्यूह)

A square matrix in which all the elements below the diagonal are zero is known as the upper triangular matrix.

मुख्य विकर्ण के नीचे के अवयव शून्य हैं। इसे 'ऊपर त्रिभुजीय आव्यूह' कहते हैं।

$$A = \begin{bmatrix} 3 & -5 & 7 \\ 0 & 4 & 0 \\ 0 & 0 & 9 \end{bmatrix}$$

⑧ Lower Triangular matrix (लोअर त्रिभुजीय आव्यूह)

A square matrix in which all the elements above the diagonal are zero is known as the upper triangular matrix.
मुख्य विकर्ण के ऊपर के अवयव शून्य हैं । इसे लोअर त्रिभुजीय आव्यूह कहते हैं।

$$A = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 4 & 0 \\ -5 & 7 & 9 \end{bmatrix}$$

⑨ Symmetric matrix (सममित आव्यूह)

If the transpose of a matrix is equal to itself, that matrix is said to be symmetric.
ऐसी वर्ग आव्यूह जिसके पंक्तियों और स्तम्भों को आपस में इस प्रकार बदला जाए कि

प्रथम पंक्ति - प्रथम स्तंभ
द्वितीय पंक्ति - द्वितीय स्तंभ

इस प्रकार नयी आव्यूह और पुरानी आव्यूह बराबर ही रहे तो ऐसी आव्यूह को सममित आव्यूह कहते हैं।

$$A = A' = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}, \quad B = B' = \begin{bmatrix} 5 & 6 & 7 \\ 6 & 3 & 2 \\ 7 & 2 & 1 \end{bmatrix}$$

⑩ Skew-symmetric matrix (विषमित आव्यूह)

A square matrix is said to be skew-symmetric if $a_{ij} = -a_{ji}$ for all i and j. In other words, we can say that matrix A is said to be skew-symmetric if transpose of matrix A is equal to negative of matrix A i.e. ($A^T = -A$)
एक वर्ग आव्यूह विषमित आव्यूह कहलाता है यदि $a_{ij} = -a_{ji}$.

$$A = \begin{bmatrix} 0 & -5 & 4 \\ 5 & 0 & -1 \\ -4 & 1 & 0 \end{bmatrix}$$

$$A^T = \begin{bmatrix} 0 & -5 & -4 \\ -5 & 0 & 1 \\ 4 & -1 & 0 \end{bmatrix}$$

$$\boxed{\therefore \quad A^T = A}$$

(11) **Row Matrix (पंक्ति आव्यूह).**

A row matrix has only one row but any number of columns. A matrix is said to be a row matrix if it has only one row.

पंक्ति आव्यूह एक ऐसा आव्यूह है जिसमें केवल एक पंक्ति होती है।

$$A = \begin{bmatrix} -1/2 & \sqrt{5} & 2 & 3 \end{bmatrix}$$

It is a row matrix of order 1×4.

(12) **Column matrix (स्तंभ आव्यूह)**

एक आव्यूह स्तंभ आव्यूह कहलाता है, यदि इसमें केवल एक स्तंभ होता है।

A Column matrix has only one column but any number of rows. A matrix is said to be a column matrix if it has only one column.

$$A = \begin{bmatrix} 0 \\ \sqrt{3} \\ -1 \\ 1/2 \end{bmatrix}$$ → It is a column matrix of order 4×1.

(13) **Rectangular Matrix (आयताकार आव्यूह)**

A matrix is said to be a rectangular matrix if the number of rows is not equal to the number of columns.

आयताकार आव्यूह एक ऐसा आव्यूह है जिसमें पंक्तियाँ स्तंभ के बराबर नहीं होती।

$$A = \begin{bmatrix} 3 & -1 & 0 \\ 3/2 & \sqrt{3}/2 & 1 \\ 4 & 3 & -1 \\ 7/2 & 2 & -5 \end{bmatrix}$$

This is a matrix of order 4×3.

Algebra of Matrices
आव्यूह बीजगणित

Addition or sum

Two matrices A and B can be added if and only if they have the same order that is the same number of rows and columns. The sum of two matrices of the same order is obtained by adding together corresponding elements of two matrices.

एक ही क्रम (order) की दो आव्यूह A और B का योग एक ऐसी आव्यूह होती है जिसके अवयव आव्यूह A और B के संगत अवयवों का योग होते हैं।

If $A = [a_{ij}]$ and $B = [b_{ij}]$, then $C = A+B$ is the matrix having a general element of the form $C_{ij} = a_{ij} + b_{ij}$

$$A = \begin{bmatrix} a_1 & b_1 \\ a_2 & b_2 \end{bmatrix}, \quad B = \begin{bmatrix} c_1 & d_1 \\ c_2 & d_2 \end{bmatrix}$$

तो $\underset{(C)}{A+B} = \begin{bmatrix} a_1+c_1 & b_1+d_1 \\ a_2+c_2 & b_2+d_2 \end{bmatrix}$

If $A = \begin{bmatrix} 3 & 2 & 5 \\ 1 & 1 & 4 \end{bmatrix}, \quad B = \begin{bmatrix} 4 & 1 & 2 \\ 5 & 3 & 0 \end{bmatrix}$

$$A+B = \begin{bmatrix} 3+4 & 2+1 & 5+2 \\ 1+5 & 1+3 & 4+0 \end{bmatrix} = \begin{bmatrix} 7 & 3 & 7 \\ 6 & 4 & 4 \end{bmatrix}$$

Hence if $A = [a_{ij}]$, $B = [b_{ij}]$ are of same order then $A+B = [C_{ij}]$

where $C_{ij} = [a_{ij}] = [b_{ij}]$

If $A = \begin{bmatrix} 1 & 5 \\ 6 & 7 \\ 8 & 9 \end{bmatrix}, \quad B = \begin{bmatrix} 10 & 2 \\ 8 & 6 \end{bmatrix}$

A+B is not defined, Since order of A and B are not the Same.

Properties of Matrix Addition

(a) Associative law = $A+B = B+A$

(b) Commutative law = $A+(B+C) = (A+B)+C$

(c) Existence of Identity = $A+0 = 0+A = A$

(d) Existence of Inverse = $A+X = 0$ then $X = -A$

Subtraction of matrices (आव्यूहों का घटाना)

Two matrices A and B can be subtracted if and only if they have the same order that is the number of column of matrix A is equal to the number of column B and the number of Row of matrix A is equal to the number of Row of B. In other words, two matrices of the same order are said to be conformable for subtraction.

जिस प्रकार से हम दो आव्यूहों को जोड़ते हैं उसी प्रकार से समान क्रम की दो आव्यूहों को घटाया भी जाता है।

If $A = [a_{ij}]$, $B = [b_{ij}]$ then

$$C = A - B$$

$$c_{ij} = a_{ij} - b_{ij}$$

$$A = \begin{bmatrix} a_1 & b_1 \\ a_2 & b_2 \end{bmatrix} \text{ तथा } B = \begin{bmatrix} c_1 & d_1 \\ c_2 & d_2 \end{bmatrix}$$

$$A - B = \begin{bmatrix} a_1 - c_1 & b_1 - d_1 \\ a_2 - c_2 & b_2 - d_2 \end{bmatrix}$$

$$A = \begin{bmatrix} 3 & 6 \\ 7 & 0 \end{bmatrix} \quad B = \begin{bmatrix} -1 & 7 \\ 8 & 4 \end{bmatrix}$$

$$A - B = \begin{bmatrix} 3+1 & 6-7 \\ 7-8 & 0-4 \end{bmatrix} = \begin{bmatrix} 4 & -1 \\ -1 & -4 \end{bmatrix}$$

Scalar Multiplication and Vector Multiplication

In matrix algebra, a simple number such as 12, −2 or .07 is called a scalar. Multiplication of a matrix by a number or scalar involves multiplication of every element of the matrix by the number. The process is called scalar multiplication because it scales the matrix up or down according to the size of the number.

किसी अदिश K और आव्यूह A का गुणनफल KA या AK एक ऐसी आव्यूह है जिसका प्रत्येक अवयव आव्यूह A के अवयव का K गुना होगा

$$K \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{bmatrix} = \begin{bmatrix} Ka_1 & Kb_1 & Kc_1 \\ Ka_2 & Kb_2 & Kc_2 \end{bmatrix}$$

$$A = \begin{bmatrix} 2 & 3 \\ 4 & 6 \\ 8 & 2 \end{bmatrix}, \text{ find } 8A = \begin{bmatrix} 16 & 24 \\ 32 & 48 \\ 64 & 16 \end{bmatrix}$$

Multiplication of a row vector A by a column vector B requires as a precondition that each vector have precisely same number of element.

$$AB = (a_{11} \times b_{11}) + (a_{12} \times b_{21}) + (a_{13} \times b_{31})$$

$$A = \begin{bmatrix} 4 & 7 & 2 & 9 \end{bmatrix}_{1\times 4} \quad B = \begin{bmatrix} 12 \\ 1 \\ 5 \\ 6 \end{bmatrix}_{4\times 1}$$

$$AB = [4(12) + 7(1) + 2\times(5) + 9\times(6)]$$

$$= [48 + 7 + 10 + 54]$$

$$= [119]$$

Multiplication of Two Matrices (आव्यूह गुणन)

Let $A = [a_{ij}]_{1\times n}$ and $B = [b_{ij}]_{n\times 1}$ be the two matrices such that the numbers of columns in A is equal to the number of rows in B. The by the product AB in that order of the $1\times n$ matrix.

$$A = [a_{11} \quad a_{12} \quad --- \quad a_{1n}]_{1\times n} \text{ and}$$

$$n\times 1 \text{ matrix } B = \begin{bmatrix} b_{11} \\ b_{12} \\ \vdots \\ b_{n1} \end{bmatrix}$$

$$AB = 1\times 1 \text{ matrix}$$

$$[a_{11} \quad a_{12} \quad --- \quad a_{1n}] \begin{bmatrix} b_{11} \\ b_{21} \\ \vdots \\ b_{n1} \end{bmatrix}$$

$$= [a_{11} b_{11} + a_{12} b_{21} + --- + a_{1n} b_{n1}]$$

$$= [\Sigma a_{ij} b_{ji}]$$

$$\begin{bmatrix} 1 & 3 & 5 \end{bmatrix} \begin{bmatrix} 2 \\ -3 \\ 4 \end{bmatrix} = [1\times 2 + 3\times(-3) + 5\times 4]$$

$$= [2 - 9 + 20] = [13]$$

दो आव्यूह A और B का गुणनफल तभी अस्तित्वमान होगा जब आव्यूह A स्तंभों की संख्या आव्यूह B में पंक्तियों की संख्या के बराबर हो

यदि A का क्रम '$m \times n$' हो तो B का क्रम '($p \times r$)' हो तो AB तभी अनुकूल होगा जब $N = P$ के हो।

यदि $A = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{bmatrix}$ $B = \begin{bmatrix} m_1 & n_1 \\ m_2 & n_2 \\ m_3 & n_3 \end{bmatrix}$

$$AB = \begin{bmatrix} a_1 \cdot m_1 + b_1 \cdot m_2 + c_1 m_3 & a_1 n_1 + b_1 n_2 + c_1 n_3 \\ a_2 m_1 + b_2 m_2 + c_2 m_3 & a_2 n_1 + b_2 n_2 + c_2 n_3 \end{bmatrix}$$

इस प्रकार यह स्पष्ट है कि यदि A का क्रम $m \times n$ तथा B का क्रम $n \times r$ हो तो AB का क्रम $m \times r$ होगा।

$$A = \begin{bmatrix} 1 & 3 \\ 2 & 1 \end{bmatrix}_{2\times2} \quad B = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 3 & 5 \end{bmatrix}_{2\times3}$$

$$[A]_{m\times n} [B]_{n\times p} = [AB]_{m\times p}$$

$$[A]_{2\times2} \; [B]_{2\times3} = [AB]_{2\times3}$$

$$= \begin{bmatrix} 1\times1+3\times2 & 1\times2+3\times3 & 1\times1+3\times5 \\ 2\times1+1\times2 & 2\times2+1\times3 & 2\times1+1\times5 \end{bmatrix}$$

$$= \begin{bmatrix} 1+6 & 2+9 & 1+15 \\ 2+2 & 4+3 & 2+5 \end{bmatrix}$$

$$= \begin{bmatrix} 7 & 11 & 16 \\ 4 & 7 & 7 \end{bmatrix}_{2\times3}$$

E If $A = \begin{bmatrix} 2 & 3 & 1 \\ 4 & 5 & 2 \end{bmatrix}_{2\times3}$, $B = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 2 \end{bmatrix}_{3\times2}$

$$AB = \begin{bmatrix} 2\times1+3\times3+1\times5 & 2\times2+3\times4+1\times2 \\ 4\times1+5\times3+2\times5 & 4\times2+5\times4+2\times2 \end{bmatrix}$$

$$AB = \begin{bmatrix} 16 & 18 \\ 29 & 32 \end{bmatrix}_{2\times2}$$

Exercise

(1) If $A = \begin{bmatrix} 2 & 0 \\ -5 & 6 \end{bmatrix}$ and, $B = \begin{bmatrix} -3 & 6 \\ 4 & 1 \end{bmatrix}$ find $A+B$.

$$A+B = \begin{bmatrix} 2+(-3) & 0+(6) \\ -5+4 & 6+1 \end{bmatrix}.$$

$$A+B = \begin{bmatrix} -1 & 6 \\ -1 & 7 \end{bmatrix}$$

(2) If $A = \begin{bmatrix} 1 & 2 & -3 \\ 0 & -1 & 2 \\ 3 & 0 & 4 \end{bmatrix}$ and $B, = \begin{bmatrix} -1 & 3 & 4 \\ 6 & 2 & 0 \\ 2 & 1 & 3 \end{bmatrix}$

$$A+B = \begin{bmatrix} 1+(-1) & 2+3 & -3+4 \\ 0+6 & -1+2 & 2+0 \\ 3+2 & 0+1 & 4+3 \end{bmatrix}$$

$$A+B = \begin{bmatrix} 0 & 5 & 1 \\ 6 & 1 & 2 \\ 5 & 1 & 7 \end{bmatrix}$$

(3) If $A = \begin{bmatrix} 1 & -3 & 0 \\ 4 & 0 & -1 \\ 0 & -2 & -4 \end{bmatrix}$ and $B, = \begin{bmatrix} -1 & 3 & -2 \\ -4 & 0 & 1 \\ 0 & 2 & 4 \end{bmatrix}$

$$A-B = \begin{bmatrix} 1-(-1) & -3-3 & 0-(-2) \\ 4-(-4) & 0-0 & -1-1 \\ 0-0 & -2-2 & -4-4 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & -6 & 2 \\ 8 & 0 & -2 \\ 0 & -4 & -8 \end{bmatrix}$$

(4) If $A = \begin{bmatrix} x & -2 & y \\ 3 & 2 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & x & z \\ x & y & 4 \end{bmatrix}$

$$A-B = \begin{bmatrix} (x-2) & (-2-x) & (y-z) \\ (3-x) & (2-y) & (3-4) \end{bmatrix} = \begin{bmatrix} (x-2) & (-2-x) & (y-z) \\ (3-x) & (2-y) & -1 \end{bmatrix}$$

(5) If $A = \begin{bmatrix} 2 & 3 & 4 \\ 5 & 6 & 7 \\ 4 & 7 & 6 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}$ find AB.

AB is conformable, hence product AB is defined and will be a 3×2 matrix.

$$C = AB = \begin{bmatrix} 2\times1+3\times3+4\times5 & 2\times2+3\times4+4\times6 \\ 5\times1+6\times3+7\times5 & 5\times2+6\times4+7\times6 \\ 4\times1+7\times3+6\times5 & 4\times2+7\times4+6\times6 \end{bmatrix}$$

$$= \begin{bmatrix} 31 & 40 \\ 58 & 76 \\ 55 & 72 \end{bmatrix}$$

(6) If $A = \begin{bmatrix} -1 & 1 & 2 \\ 2 & 3 & 4 \\ 3 & 2 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 3 & 0 \\ 0 & 1 & 0 \\ 1 & 2 & -1 \end{bmatrix}$ find whether AB is possible?

Here A is 3×3 matrix, B is 3×3 matrix.

$\therefore$ AB is defined (since number of columns of A (=3) is equal to the number of rows of B(=3).

$$AB = \begin{bmatrix} -1\times1+1\times0+2\times1 & -1\times3+1\times1+2\times2 & -1\times0+1\times0+2\times(-1) \\ 2\times1+3\times0+4\times1 & 2\times3+3\times1+4\times2 & 2\times0+3\times0+4\times(-1) \\ 3\times1+2\times0+1\times1 & 3\times3+2\times1+1\times2 & 3\times0+2\times0+1\times-1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2 & -2 \\ 6 & 17 & -4 \\ 4 & 13 & -1 \end{bmatrix}$$

(7) If $B = \begin{bmatrix} 1 & 3 & 0 \\ 0 & 1 & 0 \\ 1 & 2 & -1 \end{bmatrix}$ and $A = \begin{bmatrix} -1 & 1 & 2 \\ 2 & 3 & 4 \\ 3 & 2 & 1 \end{bmatrix}$ find BA

$$BA = \begin{bmatrix} 1\times(-1)+3\times(2)+0(3) & (1)\times1+3\times3+0\times2 & 1\times2+3\times4+0\times1 \\ 0\times(-1)+1\times(2)+0(3) & 0\times(1)+1\times3+0\times(2) & 0\times2+1\times4+0\times1 \\ 1\times(-1)+2\times2+(-1)\times3 & 1\times(1)+2\times3+2\times(-1) & 1\times2+2\times4+(-1)\times1 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & 10 & 14 \\ 2 & 3 & 4 \\ 0 & 5 & 9 \end{bmatrix}$$

⑧ If $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$. find $A^2 - 5A + 7I$.

$$A^2 = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 3\times3+(-1)\times1 & 3\times(1)+(1)(2) \\ -1\times(3)+2\times(-1) & -1\times(1)+2\times(2) \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix}$$

$$5A = 5\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix}$$

$$7I = 7\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$$

$$A^2 - 5A + 7I = \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} - \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$$

$$= \begin{bmatrix} -7 & 0 \\ 0 & -7 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

⑨ If $A = \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix}$, $B = \begin{bmatrix} a & 1 \\ b & -1 \end{bmatrix}$ and $(A+B)^2 = A^2 + B^2$ find a and b.

$$A + B = \begin{bmatrix} 1+a & 0 \\ 2+b & -2 \end{bmatrix}$$

$$(A+B)^2 = \begin{bmatrix} 1+a & 0 \\ 2+b & -2 \end{bmatrix} \begin{bmatrix} 1+a & 0 \\ 2+b & -2 \end{bmatrix} = \begin{bmatrix} (1+a)^2 & 0 \\ (2a-b+ab-2) & 4 \end{bmatrix}$$

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$$A^2 = \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$B^2 = \begin{bmatrix} a & 1 \\ b & -1 \end{bmatrix} \begin{bmatrix} a & 1 \\ b & -1 \end{bmatrix} = \begin{bmatrix} (a^2+b) & (a-1) \\ (ab-b) & (a+1) \end{bmatrix}$$

$$A^2 + B^2 = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} + \begin{bmatrix} (a^2+b) & (a-1) \\ (ab-b) & (a+1) \end{bmatrix}$$

$$= \begin{bmatrix} (a^2+b-1) & (a-1) \\ (ab-b) & b \end{bmatrix}$$

Since $(A+B)^2 = A^2 + B^2$

$$\therefore \begin{bmatrix} (1+a)^2 & 0 \\ (2a+\underset{-2}{ab}-b) & 4 \end{bmatrix} = \begin{bmatrix} (a^2+b-1) & (a-1) \\ (ab-b) & b \end{bmatrix}$$

$(a-1) = 0$ or $a = 1$, $b = 4$