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STUDENT 'T' TEST

Sir, William S. Gosset (Pen name Student) developed a Significance test and through it made Significant contribution in the theory of sampling applicable in case of small samples less than 30. ($n < 30$) when population variance in not known. The Test is commonly known as student's 't' test and is based on the 't' distribution.

Like the normal distribution, 't' distribution is symmetrical but happens to be flatter than the normal distribution. Moreover, there is a different 't' distribution for every possible sample size. As sample size gets larger, the shape of 't' distribution loses its flatness and becomes approximately equal to the normal distribution. In fact, for sample sizes of more than 30, the 't' distribution is so close to the normal distribution that we will use the normal to approximate the 't' distribution. Thus, when n is small that 't' distribution is far from normal but when n is infinite it is identical with normal distribution.

For applying t-test in context of small sample ~~the~~ the t value is calculated first of all and then compared with the table value of t at certain level of significance for given degree of freedom. If the calculated value of t exceeds that table value (say $t_{0.05}$) we infer that the difference is significant at 5% level but if t is less than the concerning table value of the 't' the difference is not treated as significant.

The 't' test is used when two conditions are fulfilled:-

I) The sample size is less than 30 i.e. when $n < 30$

II) The popn S.D (σ_p) must be unknown.

$SE_{\bar{x}_1 - \bar{x}_2}$ = Standard Error of difference between two sample means and it is worked out as follows:-

$$SE_{\bar{x}_1-\bar{x}_2} = \sqrt{\frac{\Sigma(x_1-\bar{x}_1)^2 + \Sigma(x_2-\bar{x}_2)^2}{n_1+n_2-2}} \times \sqrt{\frac{1}{n_1}+\frac{1}{n_2}}$$

and the degree of freedom $= (n_1+n_2-2)$

(iii) To Test the significance of an observed correlation co-efficient:-

$$t = \frac{r}{\sqrt{1-r^2}} \times \sqrt{n-2}$$

here t is based on $(n-2)$ degree of freedom.

(iv) In Context of the difference Test:- difference test is applied in the case of paired data and in this context t is calculated as under:-

$$t = \frac{\bar{x}_{diff.} - 0}{\frac{\sigma_{diff}}{\sqrt{n}}} = \frac{\bar{x}_{diff.} - 0}{\sigma_{diff.}}\sqrt{n}$$

where $\bar{x}_{diff.}$ or D = Mean of the differences of sample items.

0 = the value zero on the hypothesis that there is no difference.

$\sigma_{diff.}$ = standard deviation of difference

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$$= \frac{\bar{x}-\mu}{\sqrt{\frac{\Sigma(x-\bar{x})^2}{n-1}}} \times \sqrt{n}$$

In case of small samples we can use either of the following:-

(a) Probable limits with 95% confidence level:-

$$\mu = \bar{x} \pm SE_{\bar{x}} \; (t_{0.05})$$

(b) Probable limits with 99% confidence level:-

$$\mu = \bar{x} \pm SE_{\bar{x}} \; (t_{0.01})$$

At other confidence levels the limits can be worked out in a similar manner taking the concerning table value of 't' just as we have taken $t_{0.05}$ in and (i) $t_{0.01}$ in (ii) above

- If standard deviation is calculated by formula.

$$\sigma = \sqrt{\frac{\Sigma(x-\bar{x})^2}{n}}$$

Thus the value of t can be written as follows:

$$t = \frac{\bar{x}-\mu}{\sigma}\sqrt{n-1}$$

(II) <u>To test the difference between the mean of the two samples:-</u>

$$t = \frac{|\bar{x}_1 - \bar{x}_2|}{SE_{\bar{x}_1 - \bar{x}_2}}$$

Where $\bar{x}_1$ = Mean of the sample - 1

$\bar{x}_2$ = Mean of the sample 2

In using the 't' test we assume the following:-

i) That the popn is normal or approximately normal.

ii) That the observations are independent and the samples are randomly drawn samples.

iii) That there is no measurement error.

iv) That in the case of two samples, popn. variances are regarded as equal if equality of the two population means is to be tested.

The following formula are commonly used to calculate that 't' value:-

(1) <u>To test of the significance of the Mean of a random Sample</u>

$$t = \frac{|\bar{X} - \mu|}{S.\ SE_{\bar{x}}\ \bar{X}}$$

Where, $\bar{X}$ = Mean of the sample.

μ = Mean of the universe.

$SE_{\bar{x}}$ = S.E. of mean in case of small and is worked-out as follows

$$SE_{\bar{x}} = \frac{\sigma_x}{\sqrt{n}} = \frac{\sqrt{\frac{\Sigma (X-\bar{X})^2}{n-1}}}{\sqrt{n}}$$

and the degree of freedom = $(n-1)$

The above formula stated for 't' can as well stated as under:-

$$t = \frac{|\bar{X} - \mu|}{SE_{\bar{x}}}$$

$$= \frac{|\bar{X} - \mu|}{\frac{\sqrt{\frac{\Sigma (X-\bar{X})^2}{n-1}}}{\sqrt{n}}}$$