$$s = \sqrt{\frac{\Sigma(x_1-\bar{x}_1)^2 + \Sigma(x_2-\bar{x}_2)^2}{n_1+n_2-2}}$$

$$= \sqrt{\frac{10+44}{5+7-2}} = \sqrt{\frac{54}{10}} = \sqrt{5.4} = 2.324$$

$$t = \frac{\bar{x}_1-\bar{x}_2}{s}\sqrt{\frac{n_1 n_2}{n_1+n_2}}$$

$$= \frac{12-11}{2.324}\sqrt{\frac{5\times7}{5+7}}$$

$$= \frac{1}{2.324}\times\sqrt{\frac{35}{12}} =$$

$$\frac{1}{2.324}\times 1.708 = \frac{1.708}{2.324} = 0.735$$

$$v = n_1+n_2-2 = 5+7-2 = 10$$

The calculated value of t is less than table value, the hypothesis is accepted —

Question (2) A random sample of size 16 has 53 mean. The sum of the squares of the deviation taken from mean is 135. Can this sample be regarded as taken from the popn. having 56 as a mean & obtain 95% and 99% confidence limits of the mean of the popn (for $v = 15$ $t_{0.05} = 2.13$ for $v = 15$ $t_{0.01} = 2.95$)

Soln.

Let the hypothesis be that there is no significant difference between the sample mean and hypothetical popn. mean.

Applying the 't' test:-

$$\boxed{t = \frac{\bar{X} - \mu}{s} \sqrt{n}}$$

$$\bar{x} = 53, \ \mu = 56, \ n = 16, \ \Sigma (x - \bar{x})^2 = 135$$

$$s = \sqrt{\frac{\Sigma (x - x^2)^2}{n-1}} = \sqrt{\frac{135}{15}} = 3$$

$$t = \frac{53 - 56}{3} \sqrt{16}$$

$$t = 4$$

$$v = 16 - 1 = 15 \text{ for } v = 15 \ t_{0.05} = 2.13$$

The calculated value of t is more than the table value. The hypothesis is rejected. Hence the sample has not come from a popn having 53 as mean.

95% confidence limits of the popn mean

$$\bar{x} \pm \frac{s}{\sqrt{n}} t_{0.05}$$

$$= 53 \pm \frac{3}{\sqrt{16}} \times 2.13$$

$$= 53 \pm 1.6 = 51.4 \text{ to } 54.6$$

136	34	170
64	16	80
200	50	250

O	E	(O-E)	$(O-E)^2$	$(O-E)^2/E$
140	136	4	16	0.118
60	64	-4	16	0.250
30	34	-4	16	0.471
20	16	4	16	1.000
				$\Sigma(O-E)^2/E = 1.839$

χ^2 test and Goodness of fit

$$\chi^2 = \Sigma\left[(O-E)^2/E\right] = 1.839$$

$$v = (r-1)(c-1)$$

$$= (2-1)(2-1) = 1$$

For $v=1$, $\chi^2_{0.05} = 3.84$ given.

The calculated value of χ^2 is less than the table value

$\therefore$ The hypothesis is accepted.

Dr. N. N. Ma

SOME NUMERICAL PROBLEMS - Chi-squar Test.

Problem - (1) From the data given below about the treatment of 250 patients suffering from a disease state whether the new treatment is superior to the conventional treatment.

Number of Patients

Treatment	Favourable	Not favourable	Total
New	140	30	170
Conventional	60	20	80
	200	50	250

Given: for $v = 1$, $\chi^2_{0.05} = 3.84$

Soln. - Let the hypothesis be that there is no significant difference between the new and conventional treatment.

Applying χ^2 Test -

$$\text{Expectation } (AB) = \frac{(A)(B)}{N}$$

When A represent favourable
& B represent Not favourable.

$$= \frac{200 \times 170}{250} = \frac{34000}{250} = 136$$

$$= \frac{200 \times 80}{250} = \frac{16000}{250} = 64$$

$$= \frac{50 \times 170}{250} = \frac{8500}{250} = 34$$

$$= \frac{50 \times 80}{250} = \frac{4000}{250} = 16$$

Now the table of expected frequencies.

Dr. N. N. Mallo

Some Numerical Problems Student T-Test

Question (1) The manufacturer of a electric bulbs claims that his bulbs have a mean life of 25 months with a standard Deviation of 5 months. A random sample of 6 such bulbs gave the following values.

Life in months :- 24, 26, 30, 20, 20, 18.

Can you regard the producer's claim to be valid at 1% level of significance? (Given that the table values of the appropriate test statistic at the said level are 4.032, 3.707 and 3.499 for 5, 6 and 7 degree of freedom respectively.

Soln:

Let us take the hypothesis that there is no significant difference in the Mean life of bulbs in the sample and that of the popn. Applying t-test

$$\boxed{t = \frac{\bar{X}-\mu}{s}\sqrt{n}}$$

X	$X-\bar{X}$ (x)	x^2
24	+1	1
26	+3	9
30	+7	49
20	-3	9
20	-3	9
18	-5	25
$\Sigma X = 138$		$\Sigma x^2 = 102$

$$\bar{X} = \frac{\Sigma X}{N} = \frac{138}{6} = 23.$$

$$s = \sqrt{\frac{\Sigma x^2}{n-1}} = \sqrt{\frac{102}{5}} = 4.517$$

$$\frac{\text{Difference}}{S.E} = \frac{23-25}{4.517} \times \sqrt{6}$$

$$= \frac{2 \times 2.449}{4.517} = 1.084$$

$v = n-1 = 5$ for $v = 5$ $t_{0.01} = 4.032$

The calculated value of t is less than the table value. The hypothesis is accepted. Hence the producer's claim is not valid at 1% level of significance.

99% confidence limits of the popn mean.

$$\bar{x} \pm \frac{s}{\sqrt{n}} t_{0.01}$$

$$= 53 \pm \frac{3}{\sqrt{16}} 2.95$$

$$= 53 \pm 2.212 \text{ ie } 50.788 \text{ to } 55.212$$

Ans

Question (3) Two types of drugs were used on 5 and 7 patients for reducing their weight. Drug A was imported and drug B was indigenous. The decrease in the weight after using the drugs for six months was as follows.

Drug-A 10, 12, 13 11 14

Drug-B 8 9 12 14 15 10 9

is there a significant difference in the efficacy of the two drugs? If not, which drug should you buy (for $v = 10$ $t_{0.05} = 2.223$).

Soln. - Let the hypothesis be that there is no significant difference in the efficacy of the two drugs.

Apply T-Test! -

$$t = \frac{\bar{x}_1 - \bar{x}_2}{S} \sqrt{\frac{n_1 n_2}{n_1 + n_2}}$$

$\bar{x}_1$	$(x_1-\bar{x}_1)$	$(x-\bar{x}_1)^2$	x_2	$x_2-\bar{x}_2$	$(x_2-\bar{x}_2)^2$
10	-2	4	8	-3	9
12	0	0	9	-2	4
13	+1	1	12	+1	1
11	-1	1	14	+3	9
14	+2	4	15	+4	16
			10	-1	1
			9	-2	4
$\Sigma x_1 = 60$		$\Sigma(x_1-\bar{x}_1)^2 = 10$	Σx_2 77		$\Sigma(x_2-\bar{x}_2)^2 = 44$

$$\bar{x}_1 = \frac{\Sigma x_1}{n} = \frac{60}{5} = 12$$

$$\bar{x}_2 = \frac{\Sigma x_2}{n} = \frac{77}{7} = 11$$

Prob - (2) Two investigator study the income of a group of persons by method of sampling. Following result were obtained by them.

Investigator	Poor	Middle class	Well to do	Total
A	160	30	10	200
B	140	120	40	300
Total	300	150	50	500

Show that the sampling technique of atleast one of the investigator is suspected.

Soln :- Let the hypothesis be that the samples are drawn at random by the investigator and ~~they~~ there is no suspicion about the sampling technique used by the two

$$E_{11} = \frac{200}{500} \times \overset{6}{\cancel{300}} = 120$$

$$E_{12} = \frac{200}{\cancel{500}} \times \overset{30}{\cancel{150}} = 60$$

$$E_{13} = \frac{\overset{4}{\cancel{200}}}{\cancel{500}} \times 5\cancel{0} = 20$$

Similarly

$$\frac{\overset{60}{\cancel{300}}}{\cancel{500}} \times 300 = 180$$

$$\frac{300}{\cancel{500}} \times \overset{30}{\cancel{150}} = 90$$

$$\frac{300}{\cancel{500}} \times \overset{10}{\cancel{50}} = 30$$

5-(4) Weights in kg of 10 students are given below :-

38, 40, 45, 53, 47, 43, 55, 48, 52, 49

Given that - variance of the distribution of weight of all 20

for $v = 9$ $\quad \chi^2_{0.05} = 16.92 \quad \chi^2_{0.01} = 21.67$

for $v = 10$ $\quad \chi^2_{0.05} = 18.31 \quad \chi^2_{0.01} = 23.21$

Soln.

Applying χ^2 - Test

X	$X-\bar{X}$	$(X-\bar{X})^2$
38	−9	81
40	−7	49
45	−2	4
53	+6	36
47	0	0
43	−4	16
55	+8	64
48	+1	1
52	+5	25
49	+2	4
$\Sigma X = 470$	$\Sigma(x-\bar{x}) = 0$	$\Sigma(x-\bar{x})^2 = 280$

$$\chi^2 = \frac{\Sigma(x-\bar{x})^2}{\sigma^2}$$

$$= \frac{\overset{14}{\cancel{280}}}{\cancel{20}}$$

$$\boxed{\chi^2 = 14}$$

for $v = 10-1 = 9$, $\chi^2_{0.05} = 16.92$

The calculated values is Less than the table value, Therefore, the null hypothesis hold true and it can be concluded than popn. varia[illegible] may be 20 square kg.

$$\bar{X} = \frac{\Sigma X}{N} = \frac{470}{10} = 47$$

Calculation of chi-square

Group	(f_o) (observed freq)	(f_e) (expected)	$(f_o - f_e)$	$(f_o - f_e)^2$	$(f_o - f_e)^2/f_e$
AB	31	54	−23	529	9.796
Ab	469	446	+23	529	1.186
aB	158	162	+23	529	3.265
ab	1315	1338	−23	529	0.395

$$\chi^2 = \Sigma\left\{\frac{(f_o - f_e)^2}{f_e}\right\} = 14.642$$

∴ Degree of freedom in this case $= (r-1)(c-1)$
$= (2-1)(2-1) = 1$

The table value of χ^2 for degree of freedom at 5% level of significance is 3.841. The calculated value of χ^2 is much higher than this table value and hence the result of experiment does not support the hypothesis. We can then conclude that vaccination is effective in preventing the attack from small pox.

Prob. (3) The Table given below shows that the data obtained during outbreak of small pox:-

	Attacked	Not attacked	Total
Vaccinated	31	469	500
Not Vaccinated	185	1315	1500
Total	216	1784	2000

Test the effectiveness of vaccination in preventing the attack from small pox.

Soln. Let us take the hypothesis that vaccination is not effective in preventing the attack from small pox i.e vaccination and attack are independent. On the basis of this hypothesis the expected frequency corresponding to the number of person vaccinated and attacked would be -

$$\text{Expectation of } (AB) = \frac{(A) \times (B)}{N}$$

When A represents vaccination and B represents attack.

$$\therefore (A) = 500$$
$$(B) = 216$$
$$N = 2000$$

$$\text{Expectation of } (AB) = \frac{500 \times 216}{2000} = 54$$

Now using the expectation of (AB) we can write the table of expected value as follows.

	Attacked B	Not attacked b	Total
Vaccinated A	$(AB) = 54$	$(Ab) = 446$	500
Not vaccinated a	$(aB) = 162$	$(ab) = 1338$	1500
Total	216	1784	2000

120	60	20	200
180	90	30	300
300	150	50	500

Now χ^2 test and Goodness of Fit

O	E	O-E	$(O-E)^2$	$(O-E)^2/E$
160	120	40	1600	13.333
140	180	-40	1600	8.888
30	60	-30	900	15.000
120	90	30	900	10.000
10	20	-10	100	5.000
40	30	10	100	3.333

$\Sigma(O-E)^2/E = 55.554$

$$\chi^2 = \Sigma\left[(O-E)^2/E\right] = 55.554$$

$$\nu = (r-1)(c-1)$$

$$= (2-1)(3-1) = 2$$

For $\nu = 2$, $\chi^2_{0.05} = 5.991$ (given)

Thus, the calculated value of χ^2 is greater than the table value, therefore, the null hypothesis is rejected.