ELASTICITY

General properties of matter
- Elasticity
- Flow of fluid
- Surface tension

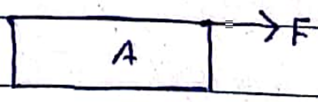
Y(~~mo~~ Young's modulas), K(Bulk modulus)
(length) (volume)

η(modulas of rigidity), σ(Poission's ratio)
length – Increases
diameter – Decreases

* Stress – $\frac{\text{Deforming force}}{\text{area}}$
* Strain – $\frac{\text{change in configuration}}{\text{original configuration}}$

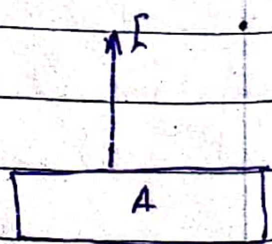
Unit – N/m², Pascal.

i) Tangential stress :-



F/A along parallel to the surface of the body

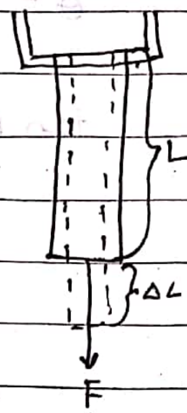
ii) Normal stress :-



F/A perpendicular to the surface of the body.

Types of Strain

1) logitudinal strain



$$= \frac{\text{Change in length}}{\text{Original length}}$$

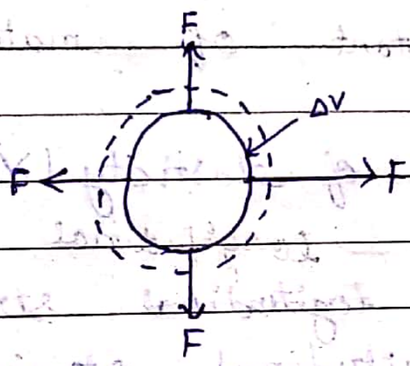
$$= \frac{\Delta L}{L}$$

2) Volumetric strain

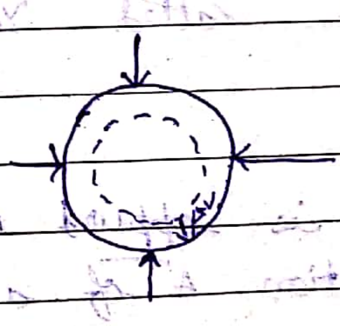
$$= \frac{\text{Change in Volume}}{\text{Original volume}}$$

$$= \frac{\Delta V}{V}$$

* Extension



* Compression



3) Shearing strain

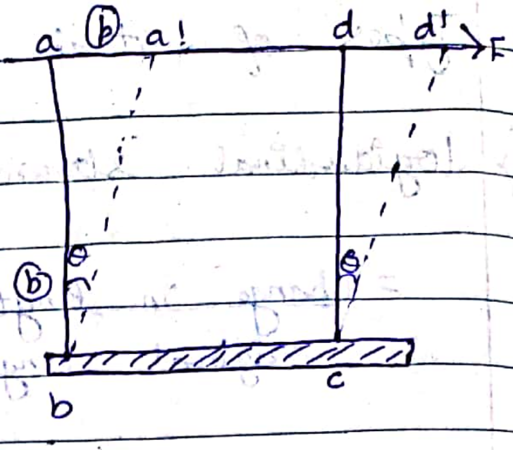
If θ be the small angle

$\angle \theta \simeq \sin\theta$

$\theta \simeq \tan\theta$

$\tan\theta = \frac{aa'}{ab}$

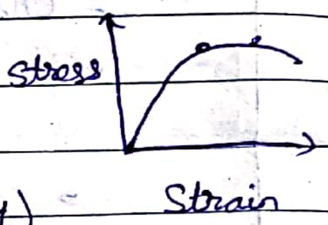
$= \frac{dd'}{cd}$



<u>Elastic Limit</u>

within elastic limit, stress is directly proportional to strain

Stress $\propto$ strain



$\frac{\text{Stress}}{\text{Strain}} = E$ (coefficient of elasticity)

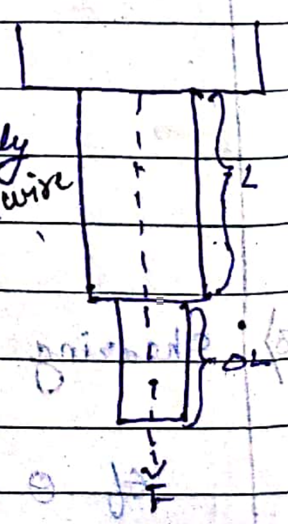
S.Q

Q) Define Elastic Constant of material.

1) Young's modulus of elasticity (Y):-

The ratio of ~~logitudinal~~ longitudinal stress to the corresponding logitudinal strain with the elastic limit is called Young modulus of the material.

If a force is applied normally to the cross-section A of a ~~wire~~ wire of length l and length of wire increased by Δl, then



$$Y = \frac{\text{longitudinal stress } (F/A)}{\text{longitudinal strain } (\Delta L/L)}$$

$$= \frac{F}{A} \cdot \left(\frac{L}{\Delta L}\right)$$

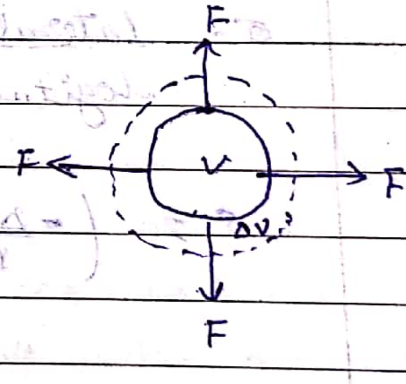
S.I unit — N/m^2

2) Bulk modulus of elasticity (K) :-

The ratio of normal stress to the volume strain within the elasticity is called Bulk modulus of the material.

If a force F is applied uniformly and normally on a total surface A of the body causing a change in volume ΔV, in its original volume V, then

$$K = \frac{\text{Normal stress } (F/A)}{\text{Volume strain } (\Delta V/V)}$$



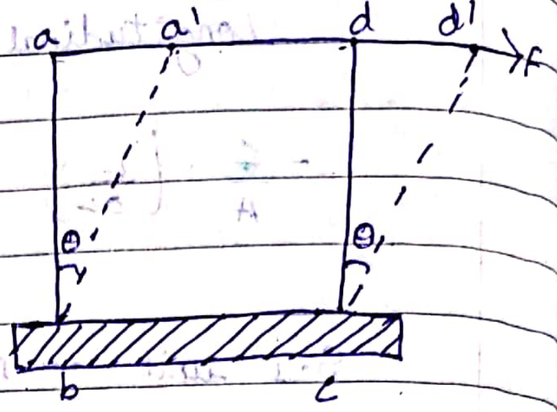
$$= \left(\frac{F}{A}\right)\left(\frac{V}{\Delta V}\right)$$

S.I unit — N/m^2

3) Modulus of rigidity (η) :-

The ratio of shearing stress to the shearing strain (θ) within the elastic limit is called modulus of rigidity of the material

$\eta = \dfrac{\text{Shearing stress } (F/A)}{\text{Shearing strain } (\theta)}$



$\eta = \dfrac{F}{A\theta}$

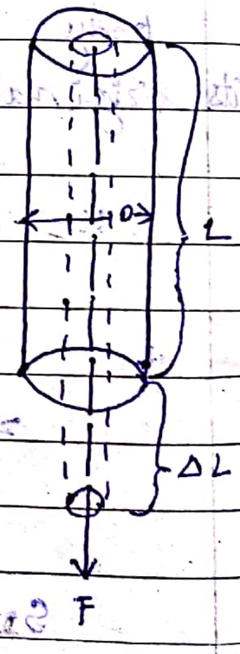
S.I unit - N/m^2

4) Poission's Ratio (σ):-

Within elastic limit the ratio of lateral stress to the longitudinal strain is constant of the material is known as Poission's ratio.

$\sigma = \dfrac{\text{lateral strain } (-\Delta D/D)}{\text{logitudinal strain } (\Delta L/L)}$

$\sigma = \left(\dfrac{-\Delta D}{D}\right)\left(\dfrac{L}{\Delta L}\right)$



It has no unit.

The ratio of lateral strain to the longitudinal strain, within elastic limit is constant of the material is known as Poisson's ratio.

6 marks

Q·1) Establish the relation among Y, K and σ (poisson's ratio)

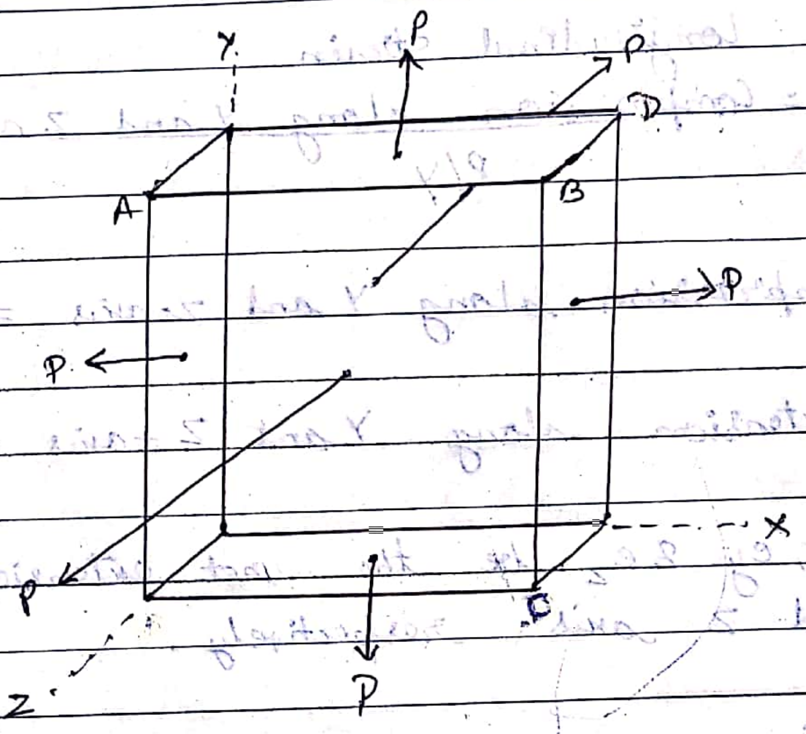
$$\boxed{Y = 3K(1-2\sigma)}$$

6 marks

Q·2) Establish the relation among Y, η (modulus of rigidity) and σ.

$$Y = 2\eta(1+\sigma)$$

1)



let us consider a cube of unit side. let a force P act normally outwards on each of its six faces. Each force produces extension in its own direction and compressions in perpendicular direction. Thus, force acting perpendicular to x-axis produce extension along x-axis and compression along Y and Z-axis.

length of ~~east~~ each side = 1
Its surface area = $1^2 = 1$

Initial Volume $= 1^3 = 1$

$$Y = \frac{\text{stress}}{\text{Longitudinal Strain}} = \frac{(P/A)}{} = \frac{P/1}{\text{Extension along x-axis}}$$

$\therefore$ Extension along X-axis $= \frac{P}{Y}$

$$\sigma = \frac{\text{lateral strain}}{\text{longitudinal strain}} = \frac{\text{Compression along Y and Z axis}}{P/Y}$$

$\therefore$ Compression along Y and z-axis $= \sigma \frac{P}{Y}$

i.e Extension along Y and Z-axis $= -\sigma \frac{P}{Y}$

let e_x, e_y & e_z be the net extension along X, Y and Z axis respectively.

$$e_x = \frac{P}{Y} - \sigma\frac{P}{Y} - \sigma\frac{P}{Y}$$

$$= \frac{P}{Y}(1 - \sigma - \sigma)$$

$$= \frac{P}{Y}(1 - 2\sigma)$$

Similarly, $e_y = \frac{P}{Y}(1 - 2\sigma)$

$$e_z = \frac{P}{Y}(1 - 2\sigma)$$

Thus, each side of the cube becomes

$$\left\{1+\frac{P}{Y}(1-2\sigma)\right\}$$

Volume of cube becomes $\left\{1+\frac{P}{Y}(1-2\sigma)\right\}^3$

[Using binomial theorem, $(1+x)^n = 1+nx$] (if $x \ll 1$)

∴ volume of cube $= 1+\frac{3P}{Y}(1-2\sigma)$

change in volume $= \cancel{1}+\frac{3P}{Y}(1-2\sigma)+\cancel{(-1)}$

$$= \frac{3P}{Y}(1-2\sigma)$$

Bulk modulus of ~~rigidity~~ Elasticity, $K = \frac{\text{Normal stress}}{\text{volume strain}}$

Or, $K = \dfrac{P/1}{\frac{3P}{Y}\frac{(1-2\sigma)}{1}}$

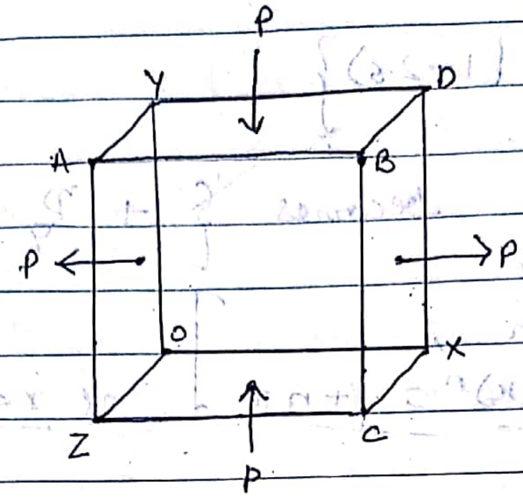
$$K = \frac{P}{1} \times \frac{Y}{3P(1-2\sigma)}$$

Or, $3K = (1-2\sigma)Y$

Or, $\boxed{Y = 3K(1-2\sigma)}$

proved

2) Relation between Y, η and σ:



Let us apply compressional stresses on the face parallel to Y-axis and equal extensional stress on the faces parallel to x-axis.

Thus, extensional stress P parallel to x-axis will produce extension $\frac{P}{Y}$ along x-axis and compression $\frac{\sigma P}{Y}$ along each of Y and Z axis. Similarly compressional stress P parallel to Y-axis will produce compression $\frac{P}{Y}$ along Y-axis and extensions $\frac{\sigma P}{Y}$ along each of x and z axis.

The net extension e_x, e_y and e_z

along x, Y and z-axis:

$$e_x = \frac{P}{Y} + \frac{\sigma P}{Y} = \frac{P}{Y}(1+\sigma)$$

$$e_y = -\frac{P}{Y} - \frac{\sigma P}{Y} = -\frac{P}{Y}(1+\sigma)$$

$$e_z = \frac{\sigma P}{Y} - \frac{\sigma P}{Y} = 0$$

The sum of extension and compression acti perpendicular to each other are equivalent to Shear.

i.e $e_x + e_y = \theta$

$$\frac{P}{Y}(1+\sigma) + \frac{P}{Y}(1+\sigma) = \theta$$

$$\frac{2P}{Y}(1+\sigma) = \theta$$

Hence modulus of rigidity, $\eta = \frac{\text{Stress}}{\text{Shearing Strai}}$

$$= \frac{P/1}{\theta}$$

$$\eta = \frac{P}{\theta}$$

$$= \frac{P}{\frac{2P}{Y}(1+\sigma)}$$

$$= \frac{P \cdot Y}{2P(1+\sigma)}$$

$$= \frac{Y}{2(1+\sigma)}$$

$$2\eta(1+\sigma) = Y$$

$$\boxed{2\eta(1+\sigma) = Y}$$

proved