Q. State and prove stephan Boltzmann law of black body radiation. Describe an experiment to verify it.

Ans **Statement:** The rate of emission of radiant energy by unit area of a perfectly black body is directly proportional to the fourth power of its absolute temperature

i.e $E \propto T^4$

$E = \sigma T^4$

where σ is called stephan's constant. Stephan law is true only for a perfectly black body. If the body is not perfectly black and its emissivity is e then

$E = e\sigma T^4$

e varies between 0 and 1, depending on the nature of the surface. For a perfectly black body e=1.

Let a perfectly black body of temperature T_1 is is surrounded by a wall of temperature T_2 Then rate of loss of radiant energy per unit area of the surface

$$E \propto (T_1^4 - T_2^4)$$

or $$E = \sigma (T_1^4 - T_2^4) \quad \text{——(iii)}$$

This law is Called stephan - Boltzmann law. If the body has an emmisivity e then

$$E = e\sigma (T_1^4 - T_2^4) \quad \text{——(iv)}$$

Proof of stephan Boltzmann law (Thermodynamic Consideration):-

From maxwall's thermodynamical relation

$$\left(\frac{\partial s}{\partial v}\right)_T = \left(\frac{\partial P}{\partial T}\right)_V$$

or $$T\left(\frac{\partial s}{\partial v}\right)_T = T\left(\frac{\partial P}{\partial T}\right)_V$$

or $$\left(\frac{\partial Q}{\partial v}\right)_T = T\left(\frac{\partial P}{\partial T}\right)_V \qquad \because \partial s = \frac{\partial Q}{T}$$

or $$\left(\frac{\partial U + P\partial V}{\partial V}\right)_T = T\left(\frac{\partial P}{\partial T}\right)_V$$

or $$\left(\frac{\partial U}{\partial V}\right)_T + P = T\left(\frac{\partial P}{\partial T}\right)_V$$

or $$\left(\frac{\partial U}{\partial V}\right)_T = T\left(\frac{\partial P}{\partial T}\right)_V - P \quad \text{——(i)}$$

Let energy density of radiation inside the cylinder of volume V is u then total energy radiation

$$U = uV$$

Pressure of radiation $P = \frac{u}{3}$

$$\therefore \left(\frac{\partial uV}{\partial V}\right)_T = T\left(\frac{\partial u/3}{\partial T}\right)_V - \frac{u}{3}$$

(u is a function of T only)

or $u\left(\frac{\partial V}{\partial V}\right)_T = \frac{T}{3}\left(\frac{\partial u}{\partial T}\right)_V - \frac{u}{3}$

or $u \times 1 = \frac{T}{3}\left(\frac{\partial u}{\partial T}\right)_V - \frac{u}{3}$

or $u + \frac{u}{3} = \frac{T}{3}\left(\frac{\partial u}{\partial T}\right)_V$

or $\frac{4}{\cancel{3}}u = \frac{T}{\cancel{3}}\left(\frac{\partial u}{\partial T}\right)_V$

or $\frac{u}{\partial u} = \frac{1}{4}\frac{T}{\partial T}$

or $\frac{\partial u}{u} = 4\frac{\partial T}{T}$

Integrating both side

$$\int\frac{\partial u}{u} = 4\int\frac{\partial T}{T}$$

$$\log_e u = 4\log_e T + \log a \text{ (Constant)}$$

or $\log_e u = \log_e T^4 + \log a$

or $\log_e u = \log_e aT^4$

or $\log_e u = \log_e aT^4 = 0$

or $\log_e\left(\frac{u}{aT^4}\right) = 0 = \log_e 1$

$$\therefore \frac{u}{aT^4} = 1$$

$\therefore u = aT^4$ —— (ii)

Let E = Energy radiated per second per unit area of a perfectly black body at absolute temperature T then

$$E = \frac{1}{4} uc$$

where c = velocity of light

$$\therefore \quad E = \frac{1}{4} aT^4 c = \frac{1}{4} acT^4$$

or $\boxed{E = \sigma T^4}$

where $\sigma = \frac{1}{4} ac = 5.6 \times 10^{-8}$ = stephan Constant

$$\therefore \quad \boxed{E \propto T^4}$$

This is stephan Boltzmann Law.

Q. Emissive Power or Radiant emittance (E)

Ans Emissive power of a body ~~at~~ a certain temperature is defined as the total thermal energy (of all wavelength) emitted per unit time per unit surface area of the body at that temperature.

Unit of emissive power $= J s^{-1} m^{-2}$

Q. Absorptive Power

Ans The absorptive power of a body at a given temperature and for a given wavelength is defined as the ratio of the radiant energy absorbed per second by unit surface area of the body to the total energy falling per second on the some area.

Q. Explain Black body.

Ans A body which absorbs all the radiation on it is called a perfectly black body. As it neither reflects nor transmitts any radiation. it appears black.

when a black body is heated it emits radiation of all wavelength. In actual practise there is no such body which behaves like a perfect black body. The nearest - approach to a perfect black body is a surface coated with lamp black or platinum black. which can be absorbs 98%. of the incident radiation falling upon it.

Q. Write Planck's Quantum Postulates. Deduce Planck's Quantum radiation Law.

Ans i) A black body radiation chamber is filled with radiations. The chamber is also filled with simple harmonic oscillator (resonator or energy emitter) of molecular dimension. These oscillators are known as plank's oscillators. The oscillators can vibrate with all possible frequencies. The oscillators has only one degrees of freedom.

ii) The oscillators cannot radiates or absorbs energy continueously. The oscillators emits or absorbs the energy in the form of packet called photon.

The energy of each photon $= h\nu$

where $h = 6.62 \times 10^{-34}$ Js

ν = frequency of radiation

Derivation of Planck's Law of radiation

Let N = Total number of oscillators

E = Total energy

$\therefore$ Average energy per oscillators

$$\bar{E} = \frac{E}{N}$$

Let N_0 = Number of oscillator having energies zero

N_1 = Number of oscillators having energy $h\nu$

N_n = No of oscillators having energies $nh\nu$

The relative probability that an oscillator has the energy $h\nu$ at temperature T $= e^{-h\nu/KT}$

where $K = 1.38 \times 10^{-23} JK^{-1}$ Boltzmann Constant

Hence $N_n = N_0 e^{-nh\nu/KT}$

The total number of oscillators

$$N = N_0 + N_1 + N_2 + N_3 + \dots + N_n$$

or $$N = N_0 + N_0 e^{-h\nu/KT} + N_0 e^{-2h\nu/KT} + N_0 e^{-3h\nu/KT} + \dots + N_0 e^{-nh\nu/KT}$$

or $$N = N_0 \left[1 + e^{-h\nu/KT} + e^{-2h\nu/KT} + e^{-3h\nu/KT} + \dots \right]$$

or $$N = N_0 \cdot \left\{ \frac{1}{1 - e^{-h\nu/KT}} \right\}$$ Sum of G.P series

or $$N = \frac{N_0}{1 - e^{-h\nu/KT}} \quad \text{—— 1}$$

Total energy of oscillators :

$$E = (N_0 \times 0) + (N_1 \times h\nu) + (N_2 \times 2h\nu) + (N_3 \times 3h\nu) + \dots$$

$$E = 0 + (N_0 e^{-h\nu/KT} \times h\nu) + (N_0 e^{-2h\nu/KT} \cdot 2h\nu) + (N_0 e^{-3h\nu/KT} 3h\nu) + \dots$$

or $$E = (N_0 e^{-h\nu/KT} \cdot h\nu) + N_0 e^{-2h\nu/KT} \cdot 2h\nu + N_0 e^{-3h\nu/KT} 3h\nu + \dots$$

$$E = N_0 h\nu \, e^{-h\nu/KT}\left[1 + 2e^{-h\nu/KT} + 3e^{-2h\nu/KT} + \cdots\right]$$

or $$E = N_0 h\nu \, e^{-h\nu/KT}\left[\frac{1}{\left(1-e^{-h\nu/KT}\right)^2}\right] \quad \text{—— (ii)}$$

$\because 1+2x+3x^2+\cdots = \frac{1}{(1-x)^2}$

Hence average energy of oscillator

$$\bar{E} = \frac{E}{N}$$

or $$\bar{E} = \frac{\cancel{N_0} h\nu \, e^{-h\nu/KT}}{\left(1-e^{-h\nu/KT}\right)^{\cancel{2}}} \times \left(\frac{\cancel{1-e^{-h\nu/KT}}}{\cancel{N_0}}\right)$$

or $$\bar{E} = \frac{h\nu \, e^{-h\nu/KT}}{\left(1-e^{-h\nu/KT}\right)}$$

Dividing Nom and Denom. by $e^{-h\nu/KT}$ we get

$$\bar{E} = \frac{h\nu}{\left(\frac{1}{e^{-h\nu/KT}} - 1\right)}$$

$$\bar{E} = \frac{h\nu}{e^{h\nu/KT}-1} \quad \text{—— (iii)}$$

The energy density of radiation $(E_\nu d\nu)$ of frequencies between ν and $\nu + d\nu$

$$E_\nu d\nu = \frac{8\pi\nu^2}{c^3} d\nu \, \bar{E}$$

or $$E_\nu d\nu = \frac{8\pi\nu^2}{c^3} d\nu \cdot \frac{h\nu}{e^{h\nu/KT}-1}$$

or $$\boxed{E_\nu d\nu = \frac{8\pi h}{c^3} \frac{\nu^3 d\nu}{e^{h\nu/KT}-1}} \quad \text{—(iv)}$$

This is planck's radiation formula in terms of frequency

$\because \quad c = \nu\lambda \quad \therefore \nu = \frac{c}{\lambda}$

$\therefore d\nu = \frac{-c}{\lambda^2} d\lambda$

$$\therefore |E_\lambda d\lambda| = \frac{8\pi h}{\cancel{c^3}} \cdot \frac{\cancel{c^3}}{\lambda^3} \frac{c}{\lambda^2} \frac{d\lambda}{e^{hc/\lambda KT}-1}$$

$$\boxed{|E_\lambda d\lambda| = \frac{8\pi hc}{\lambda^5} \frac{d\lambda}{(e^{hc/\lambda KT}-1)}} \quad \text{—(V)}$$

This is planck's radiation formula in terms of wave length.

Q. Prove that planck's Law reduces to
i) Wein's Low (Weins distribution Low) for shorter wavelength
ii) Rayleigh - Jeans law for Longer wavelength

Ans i) Wein's Low from Planck's law

Planck's radiation law is

$$E_\lambda d\lambda = \frac{8\pi hc}{\lambda^5} \frac{d\lambda}{e^{hc/\lambda KT}-1}$$

For shorter wavelength $e^{hc/\lambda KT}$ becomes large as compared to unity.

Hence planck's law reduces to

$$E_\lambda d\lambda = \frac{8\pi hc}{\lambda^5} \frac{d\lambda}{e^{hc/\lambda KT}}$$

$$\therefore E_\lambda d\lambda = \frac{8\pi hc}{\lambda^5} e^{-hc/\lambda KT} d\lambda$$

This is weein's Law.

ii) Rayleigh Jeans law from planck radio

Ans Planck's radiation Law is

$$\because E_\lambda d\lambda = \frac{8\pi hc}{\lambda^5} \frac{d\lambda}{e^{hc/\lambda KT}-1}$$

For longer wavelength, $e^{hc/\lambda KT}$ can be equal to $1 + \frac{hc}{\lambda KT}$

Hence planck's law reduces to

$$E_\lambda d\lambda = \frac{8\pi hc}{\lambda^5} \frac{d\lambda}{\cancel{1}+\frac{hc}{\lambda KT}\cancel{-1}}$$

$$\text{or } E_\lambda d\lambda = \frac{8\pi hc}{\lambda^5} \cdot \frac{d\lambda}{hc/\lambda KT}$$

$$\text{or } E_\lambda d\lambda = \frac{8\pi \cancel{hc}}{\lambda^5} \times \frac{\lambda KT\, d\lambda}{\cancel{hc}}$$

$$\text{or } \boxed{E_\lambda d\lambda = \frac{8\pi KT}{\lambda^4} d\lambda}$$

This is Rayleigh Jeons law

Unit of stephan's Constant $= J m^{-2} s^{-1} K^{-4}$

III) Derive stephan's law from planck's law of radiation.

Ans Planck's radiation formula in terms of frequency is given by

$$E_\nu d\nu = \frac{8\pi h}{c^3} \frac{\nu^3 d\nu}{e^{h\nu/KT} - 1}$$

Hence total radiant energy over fall frequencies.

$$E = \int_0^\infty E_\nu d\nu = \frac{8\pi h}{c^3} \int_0^\infty \frac{\nu^3 d\nu}{e^{h\nu/KT} - 1} \quad \text{---(i)}$$

putting $\frac{h\nu}{KT} = x$ $\therefore \nu = \frac{KT}{h} x$

$$d\nu = \frac{KT}{h} dx$$

Hence equation ① becomes

$$E = \frac{8\pi \cancel{h}}{c^3} \int_0^\infty \frac{K^3 T^3 x^3}{h^3 (e^x - 1)} \cdot \frac{KT\, dx}{\cancel{h}}$$

or $$E = \frac{8\pi K^4 T^4}{c^3 h^3} \int_0^\infty \frac{x^3\, dx}{(e^x - 1)}$$

But $$\int_0^\infty \frac{x^3 dx}{e^x - 1} = \frac{\pi^4}{15}$$

$$\therefore E = \frac{8\pi K^4 T^4}{h^3 c^3} \cdot \frac{\pi^4}{15}$$

$$E = \frac{8\pi^5 K^4 T^4}{15 h^3 c^3}$$

$$E = a T^4$$

where $a = \frac{8\pi^5 K^4}{15 c^3 h^3}$

But $\frac{ac}{4} = \sigma$ (stephen Constant)

$$\therefore \boxed{E = \sigma T^4}$$

This is stephan Low.

Minimum size in Quantum mechanics
In quantum mechanics, the value of ho is determined by Heisenberg's uncertanity principle which states that $dx \cdot dp_x \geq h$ where $h = 6.6 \times 10^{-34}$ Js = Planck's Constant

Simillarly $dy \cdot dp_y \geq h$

and $dz \, dp_z \geq h$

Hence $d\tau = (dx \cdot dp_x)(dy \cdot dp_y) \cdot (dz \cdot dp_z)$

$= dx\,dy\,dz\,dp_x\,dp_y\,dp_z \geq h^3$

Hence minimum size of cell in Quantum mechanical system is h^3.