Q. Write short notes on 'phase space'.

Ans In classical mechanics, the state of a single particle can be completly specified by position co-ordinate (x, y, z) and momentum co-ordinate (p_x, p_y, p_z).

A combination of the position space and momentum space is known as phase space. Therefore phase space has six dimension.

A point in phase space is completly specified by six co-ordinate x, y, z, p_x, p_y and p_z.

A small volume element in phase space is denoted by $d\tau$ and $d\tau = dx\,dy\,dz\,dp_x\,dp_y\,dp_z$

But $dx\,dy\,dz$ gives a small volume element dv in position space and $dp_x\,dp_y\,dp_z$ gives small volume element $d\Gamma$ in momentum space

$$\therefore \quad d\tau = dv\, d\Gamma$$

Hence a volume element $d\tau$ in phase space is the product of volume element dv in position space and volume element $d\Gamma$ in momentum space

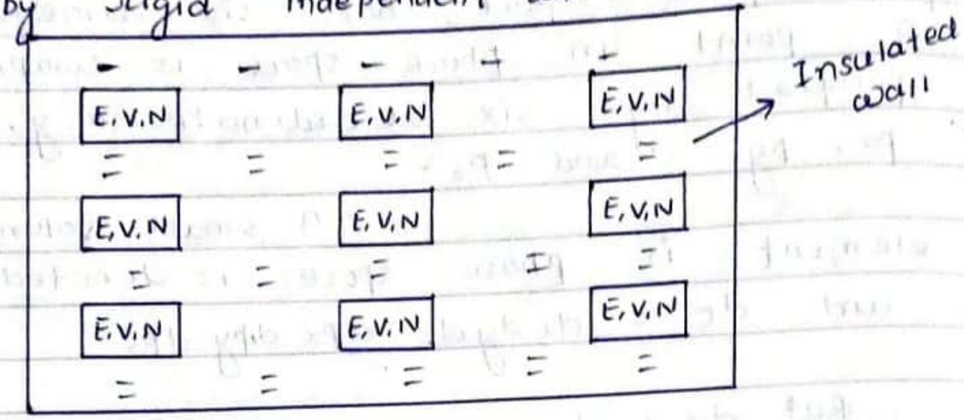
Q Explain the types of ensembles.

Ans An ensembles is defined as the collection of a large number of microscopically identical but essentially independent system.

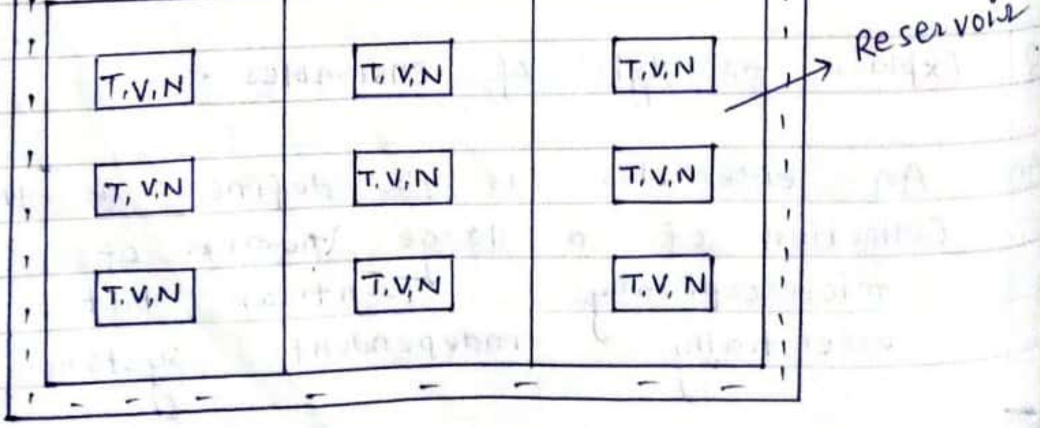
impermeable अभेद

There are three types of ensembles

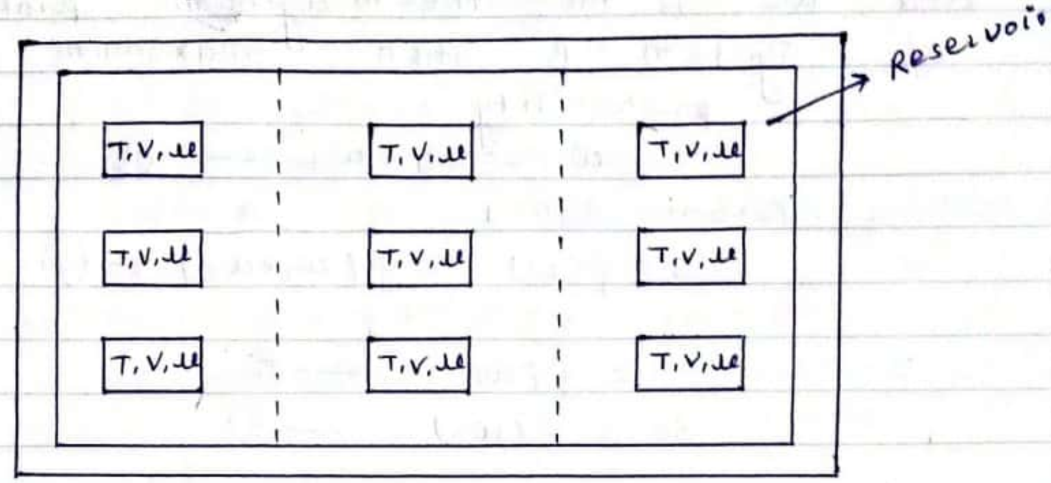
i) **Micro connonical Ensembles** : It is the collection of large number of essentially independent system which have the same value of energy (E), Volume (V) and number of particle (N). The individual system of a micro connonical ensembles are sperated by rigid independent and well-insulated wall.



II) **Connonical Ensembles** → It is the collection of a large number of independent system having the some temperature (T), Volume (V) and same number of identical particle (N) The individual system of a connonical ensembles are seperated by rigid, impermeable but conducting wall



iii) Grond Connonical Ensembles :- It is the collection of a large number of essentially independent system having same temperature (T), volume (V) and chemical potential (μ). The individual system of a grond connonical ensembles are seperated by rigid. permeable and conducting wall.



O.P Q. Deduce relation between entropy and thermodynamic probability

OR

Deduce Boltzmann's entropy relation.

Ans Entropy is a thermodynamical property and probability is a statistical quantity.

Boltzmann assumed that entropy s of a physical system in a definite state is a function of maximum probability ω of that state

Hence $s = f(\omega)$ —— 1

Let us Consider two completly independent system A and B having entropies s_1 and s_2. Since entropy is an exténsive (i.e additive) Quantity, the entropy of two system

$$S = S_1 + S_2 \quad — ②$$

Let ω_1 is the thermodynamic probability of system A and ω_2 is the thermodynamic probability of system B then maximum probability

$$\omega = \omega_1 \times \omega_2 \quad — ③.$$

From Eqn 1

$$S = f(\omega) = f(\omega_1 \times \omega_2) \quad — ④$$

$$S_1 = f(\omega_1) \quad — ⑤$$

$$S_2 = f(\omega_2) \quad — ⑥$$

Substituting Equation ④, ⑤ and ⑥ in Equation ②, weget

$$f(\omega_1 \times \omega_2) = f(\omega_1) + f(\omega_2) \quad — ⑦$$

Defferentiating partially w.r to ω_1 and ω_2, weget

$$\omega_2 f'(\omega_1 \times \omega_2) = f'(\omega_1) \quad — ⑧$$

$$\omega_1 f'(\omega_1 \times \omega_2) = f'(\omega_2) \quad — ⑨$$

Dividing Equation ⑧ by Eqn ⑨

$$\frac{\omega_2 \cancel{f'(\omega_1 \times \omega_2)}}{\omega_1 \cancel{f'(\omega_1 \times \omega_2)}} = \frac{f'(\omega_1)}{f'(\omega_2)}$$

or $$\frac{\omega_2}{\omega_1} = \frac{f'(\omega_1)}{f'(\omega_2)}$$

or $\frac{f'(\omega_1)}{f'(\omega_2)} = \frac{\omega_2}{\omega_1}$

or $\omega_1 f'(\omega_1) = \omega_2 f'(\omega_2) = k$

where k is Constant

$\therefore f'(\omega_1) = \frac{k}{\omega_1}$, $f'(\omega_2) = \frac{k}{\omega_2}$

Integrating it weget

$f(\omega_1) = k \log \omega_1 + C_1$

& $f(\omega_2) = k \log \omega_2 + C_2$

In general

$f(\omega) = k \log \omega + C$

where C is Contant integration

Hence

$S = k \log \omega + C$ —— ⑩

At absolute zero temperature

$T = 0K$

$\therefore$ entropy $0 = k \log 1 + C$

or $0 = 0 + C$

$\therefore C = 0$

Hence from Eqn ⑩

$\boxed{S = k \log \omega}$ —— 11

Equation 11 is the relation between entropy and thermodynamic probabitiy and is called Boltzmann entropy relation.

Hence Boltzmann's entropy relation states that the entropy of a system is proportional to the logarithmic of thermodynamic probability of that system.

Compartment - जगह (डिब्बा)

Q Explain Micro state and macro state

Ans Distinct arrangement of distinguishable particle of a system is called micro state. For example if three distinguisable particle are distributed in two compartment then 8 distinct arrangement are possible as given below

[abc, o], [ab, c], [ac, b], [bc. a], [a, bc], [b, oc], [c, ab], [o, abc].

i.e. there exist 8 arrangement of particles in micro state of the system. thus a micro state is distribution of distinguishable particle among them selves.

Macro state : The arrangement of the particle of the system without distinguished them from one and other is called Macro state of the system. If there are three particle to be distributed in two Compartment without distinguishable among the particle then they are four possible arrangement given below

Arrangement I - [abc, o]
Arrangement II - [ab, c] or [ac. b] or [bc, a]
Arrangement III - [a, bc] or [b, oc] or [c, ab]
Arrangement IV - [o, abc]

If one of the arrangement is called macro state of the system then number of particle in two compartment defind the micro state of the system.

Thermodynamical Probability : The number of microstate in a given macrostate is called Thermodynamical probability. It is denoted by W. For example If three distinguished particle are distributed in two compartment then the macro state (1,2) has three microstate namely [ac, b], [ab, c], [bc, a]. Therefore the thermodynamical probability of macro state (1,2) is 3 we may denoted denoted it by

$$W(1,2) = 3$$
$$W(2,1) = 3$$
$$W(3,0) = 1$$
$$W(0,3) = 1$$

Relation between theromodynomical probability and probability of micro and macro state.

The probability of Each microstate of the system is given by

$$P = \frac{1}{\text{Totot number of micro state of the system}} \quad \text{—(i)}$$

Probability of macro state is given by

$$P = \frac{\text{Number of micro state in given macro state}}{\text{Total number of macrostate of the system}}$$

But number of microstate of the system is called thermodynamical probability W

$$\therefore \; P = \frac{W}{\text{Total number of micro state of the system}} \quad \text{—(ii)}$$

Dividing Equation ⑪ by Equation (i) weget

$$\frac{P}{p} = \frac{W}{\cancel{\text{Total no of microstate of the system}}} \times \frac{\cancel{\text{Total no of microstate of the system}}}{1}$$

$$\Rightarrow \frac{P}{p} = W$$

$$\therefore \boxed{P = Wp}$$

This is the relation of thermodynamic probability with micro and macro state of the system.

Q. Give the theory of fermi energy in a metal (Conductor).

Ans An atom in a metal has weakly. bound electron in its outer most orbit (shell). When a metallic solid is formed its atoms are closely packed together. since the atoms are very close together hence there arises forces between them due to interaction of their charge distribution. These forces are called interatomic force. As a result of these forces the weakly bound electrons are set free from their individual atom. These electrons move freely like molecule of gas within the boundary of the solid. Such assembly electrons are known as free electron gas. Thus a metallic solid consist of free electron gas.

Let us consider an electron gas having n free electron in a conductor at absolute temperature T. V being the volume of conductor. Let the total energy of electron be u. This energy is distributed among all the electron according to Fermi distribution law is given by

$$n_i = \frac{g_i!}{e^{\alpha} e^{u_i/KT} + 1} \quad \text{—— (i)}$$

If we consider continueous distribution of energy among electron then we may replace u_i by u

Let $n(u)$ be the number of electron and $g(u)$ be the number of phase space shell lying between the interval u and $u+du$ and number of phase space and number of electron be $n(u)\,du$ then eqn (i) reduces to

$$n(u)\,du = \frac{g(u)\,du}{e^{\alpha} e^{u/KT} + 1} \quad \text{—— (2)}$$

The number of phase space cells having momentum p and p+dp is given by

$$g(p)\,dp = \frac{4\pi V}{h_0^3} p^2 dp \quad \text{—— ③}$$

where V is the volume of Conductor and h_0^3 is the volume of phase space cell. Since electron obeys Fermi - Dirac statistics.

$$\therefore h_0^3 = h^3$$

Hence above equation can be written as

$$g(p)\,dp = \frac{4\pi V}{h^3} p^2 dp \quad \text{—— (4)}$$

Since electron exist in two spins states namely $+\frac{1}{2}$ and $-\frac{1}{2}$. Therefore total number of phase space cell is given by

$$g(p)\,dp = 2 \times \frac{4\pi V}{h^3} p^2 dp$$

$$g(p)\,dp = \frac{8\pi V}{h^3} p^2 dp \quad \text{—— ⑤}$$

The energy u of electron of mass m having momentum p is given by

$$u = \frac{p^2}{2m}$$

$$\therefore p^2 = 2mu$$

deff it.

$$\cancel{2}p\,dp = \cancel{2}m\,du$$

$$dp = \frac{m}{p} du \quad \text{—— [illegible]}$$

Putting the value of p^2 and dp in Equation ⑤, we get

$$g(u)\,du = \frac{8\pi V}{h^3} \cdot 2mu \cdot \frac{m}{\sqrt{2mu}} du$$

Accommodate - रहने का जगह

$$g(u)du = \frac{8\sqrt{2}\pi V}{h^3} m^{3/2} u^{1/2} du \quad \text{——} ⑥$$

From Eqn ② & ⑥ we get

$$n(u)du = \frac{8\sqrt{2}\pi V}{h^3} m^{3/2} u^{1/2} du \Big/ e^{\alpha} e^{u/KT} + 1$$

or

$$n(u)du = \left(\frac{8\sqrt{2}\pi V}{h^3}\right) \frac{m^{3/2} \cdot u^{1/2} du}{e^{\alpha} e^{u/KT} + 1} \quad \text{——} ⑦$$

Equation ⑦ is called Fermi Dirac law of distribution of energy among them.

In expression ⑦, the value of α still unknown. To calculate the value of α we first define a very important Quantity called Fermi energy.

Fermi energy :-

From Equation ⑦ it is clear that as absolute temperature T decreases the energy of electron decreases. The energy of the system as a whole also decreases but at absolute zero (0K) the energy of electron is not zero and the levels of energy feeling from lowest energy level till electron are accommodate in energy level. Since electron obeys Pauli exclusive principle, only one electron can exist in each energy level. So there is a particular energy level above which no electron exist. This energy level is Called Fermi energy level. Therefore we can say that the energy of level below which all energy levels are filled and above all the levels are vacant.

Fermion - electron

Bosson particle - proton (God Particle)

This is called Fermi energy level. The number of Fermi energy level can obtained by integrating Eqn ⑦

$$n = \int_0^{u_f} \frac{8\sqrt{2}\pi V}{h^3} m^{3/2} u^{1/2} du$$

$$= \frac{8\sqrt{2}\pi V}{h^3} m^{3/2} \int_0^{u_f} u^{1/2} du$$

$$= \frac{8\sqrt{2}\pi V}{h^3} m^{3/2} \cdot \frac{2}{3} \left[u^{3/2} \right]_0^{u_f}$$

$$= \frac{16\sqrt{2}\pi V}{3h^3} m^{3/2} u_f^{3/2}$$

$$\therefore u_f = \left[\frac{3n h^3}{16\sqrt{2}\pi V m^{3/2}} \right]^{2/3} \quad \text{—} ⑧$$

Equation ⑧ is an expression for Fermi energy.

Q. Derive Fermi - Dirac distribution law

$$n(u)du = \frac{g(u)du}{e^{\alpha} \cdot e^{u/KT} + 1}$$

Ans Let us consider an isolated system consisting of non interacting indistinguishable identical particle (Fermions) whose spin angular momentum is odd half integral multiple of $\hbar$. The system is in equilibrium at absolute temperature T, total energy u volume V and number of particle n. since the particles are indistinguishable and obey Pauli exclusive principle which states that number of particles in each shell will be zero or one. Thus number of particles in a shell is either 0 or 1. In this case number of shells may be greater than the number of particles $(g_i > n_i)$.

Let us now Consider an energy compartment u_i with g_i shells containing n_i particles. The first particles can be placed in g_i shell in g_i ways, 2nd particle can be placed by (g_i-1) ways, 3rd particle (g_i-2) ways and so. on. Thus n_i particle can be placed in g_i shells of i^{th} compartment

$$= \frac{g_i!}{(g_i-n_i)!}$$

In each of these arrangement the number of permutation of n_i particle among themselves is $n_i!$ since the particles are indistinguishable. Hence these permutation donot give independent arrangement and hence actual number of independent permutation of n_i particle in g_i shell is given by

$$\omega_i = \frac{g_i!}{n_i!\,(g_i-n_i)!}$$

The total number ω of independent ways of obtaining a distribution $(n_1, n_2, \dots n_i)$ particle among a compartment for $i = 1, 2, 3, \dots k$ is given by

$$\omega = \frac{g_1!}{n_i!(g_i-n_i)!} \times \frac{g_2!}{n_2!(g_2-n_2)!} \times \dots \frac{g_K}{n_K!(g_K-n_K)!}$$

$$\omega = \prod_{i=1}^{K} \frac{g_i!}{n_i!(g_i-n_i)!} \quad \text{———} \quad 1$$

The Condition for most probable distribution is given by

$$\frac{\partial \log\omega}{\partial n_i} - \alpha_i - \beta u_i = 0 \quad \text{———} \quad ②$$

Taking logirithmic on both side of Equation 1

$$\log\omega = \sum_{i=1}^{K}\left[\log \frac{g_i!}{n_i!(g_i-n_i)!}\right]$$

$$\text{or} \quad \log\omega = \sum_{i=1}^{K}\left[\log g_i! - \log n_i! \times (g_i-n_i)!\right.$$

$$\text{or} \quad \log\omega = \sum_{i=1}^{K}\left[\log g_i! - \log n_i! - \log (g_i-n_i\right]$$

$$\text{———} \quad ③$$

From stering formula

$$\log g_i! = g_i \log g_i - g_i$$

$$\log n_i! = n_i \log n_i - n_i$$

$$\log(g_i-n_i)! = (g_i-n_i)\log(g_i-n_i) - (g_i-n_i)$$

Putting these value in Equation 3, weget

$$\log\omega = \sum_{k=1}^{K}\Big[g\log g_i - \cancel{g_i} - n_i\log n_i + \cancel{n_i} - (g_i-n_i)\log(g_i-n_i) + \cancel{g_i} - \cancel{n_i}\Big]$$

or $$\log\omega = \sum_{i=1}^{K}\Big[g_i\log g_i - n_i\log n_i - (g_i-n_i)\log(g_i-n_i)\Big] \quad \text{—④}$$

Diff w.r.t n_i taking g_i fixed

$$\frac{\partial\log\omega}{\partial n_i} = 0 - \cancel{n_i}\times\frac{1}{\cancel{n_i}} - \log n_i\times 1 - \cancel{(g_i-n_i)}\frac{1}{\cancel{(g_i-n_i)}}(0-1) - \log(g_i-n_i)(0-1)$$

or $$\frac{\partial\log\omega}{\partial n_i} = \cancel{-1} - \log n_i \cancel{+1} + \log(g_i-n_i)$$

or $$\frac{\partial\log\omega}{\partial n_i} = \log(g_i-n_i) - \log n_i$$

or $$\frac{\partial\log\omega}{\partial n_i} = \log\frac{(g_i-n_i)}{n_i} \quad \text{—⑤}$$

Putting these value in Equation ②, weget

$$\log\frac{(g_i-n_i)}{n_i} - \alpha_i - \beta u_i = 0$$

$$\log\frac{(g_i-n_i)}{n_i} = \alpha_i + \beta u_i$$

$$\Rightarrow \frac{g_i - n_i}{n_i} = e^{\alpha_i + \beta u_i}$$

$$\Rightarrow \frac{g_i}{n_i} - 1 = e^{\alpha_i + u_i/KT} \qquad \because \beta = \frac{1}{KT}$$

$$\Rightarrow \frac{g_i}{n_i} = e^{\alpha_i + u_i/KT} + 1$$

But $n_i = n(u)\,du$

$g_i = g(u)\,du$

$u_i = u$

Hence above equation reduces to

$$\frac{g(u)\,du}{n(u)\,du} = e^{\alpha_i + u/KT} + 1$$

or
$$\frac{n(u)\,du}{g(u)\,du} = \frac{1}{e^{\alpha_i + u/KT} + 1}$$

$$\therefore \boxed{n(u)\,du = \frac{g(u)\,du}{e^{\alpha_i + u/KT} + 1}}$$

Q Deduce Planck's Law of radiation from Boss Einstein distribution law

Ans Boss Einstein distribution law is given by

$$n(u)\,du = \frac{g(u)\,du}{e^{\alpha}\, e^{u/KT} - 1}$$

$\because E = mc^2, \quad E = h\nu$

$$mc^2 = h\nu$$
$$\Rightarrow mc = \frac{h\nu}{c}$$
$$\Rightarrow p = \frac{h\nu}{c}$$

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But for photon $u = h\nu$ then about Equation reduces to

$$n(\nu)d\nu = \frac{g(\nu)d\nu}{e^{\alpha}\, e^{h\nu/kT} - 1} \quad \text{—— 1}$$

where $n(\nu)d\nu$ is the number of photon b/w frequency range ν and $\nu + d\nu$ and $g(\nu)d\nu$ is the number of phase space cells in the frequency range ν and $\nu + d\nu$

Number of phase space cell corresponding to momentum range p and $p+dp$ is given by

$$g(p)\,dp = \frac{4\pi V}{h_0^3} p^2 dp.$$

But light is transverse in nature and it has two direction of polarisation therefore number of phase space cells between momentum interval p and $p+dp$ is given by

$$g(p)\,dp = 2 \times \frac{4\pi V}{h_0^3} p^2 dp$$

For photon $h_0^3 = h^3$

$$\therefore g(p)\,dp = \frac{8\pi V}{h^3} p^2 dp \quad \text{—— ②}$$

$$p = \frac{h\nu}{c}$$

$$\therefore dp = \frac{h}{c} d\nu$$

Putting these value in Equation ②

$$g(\nu)d\nu = \frac{8\pi V}{\cancel{h^3}} \frac{\cancel{h^2}\nu^2}{c^2} \cdot \frac{\cancel{h}}{c} d\nu$$

$$g(\nu)d\nu = \frac{8\pi V}{c^3} \nu^2 d\nu \quad \text{—— ③}$$

when $\alpha \to 0$ then $e^0 = 1$

Putting this value in Equation (i)

$$n(\nu)d\nu = \frac{8\pi V}{c^3}\nu^2 d\nu \frac{1}{e^{\alpha} e^{h\nu/KT} - 1}$$

Energy density of photon

$$u(\nu)d\nu = \frac{h\nu}{V} n(\nu)d\nu$$

$$= \frac{h\nu}{\cancel{V}} \cdot \frac{8\pi \cancel{V}}{c^3} \frac{\nu^2 d\nu}{e^{\alpha} e^{h\nu/KT} - 1}$$

$$u(\nu)d\nu = \frac{8\pi h\nu^3}{c^3} \frac{1}{e^{\alpha} e^{h\nu/KT} - 1} d\nu \quad \text{—} ④$$

This is plonck's low of radiation in term of frequency

But $c = \nu\lambda$

$$\nu = \frac{c}{\lambda}$$

$$d\nu = \frac{-c}{\lambda^2} d\lambda$$

Neglecting negative sign

$$d\nu = \frac{c}{\lambda^2} d\lambda$$

putting this value in Equation ④

$$u(\lambda)\, d\lambda = \frac{8\pi h}{\cancel{c^3}} \frac{\cancel{c^3}}{\lambda^3} \frac{1}{e^{\alpha} e^{hc/\lambda KT} - 1} \cdot \frac{c}{\lambda^2} d\lambda$$

$$\Rightarrow u(\lambda)d\lambda = \frac{8\pi hc}{\lambda^5} \frac{1}{e^{\alpha} e^{hc/\lambda KT} - 1} d\lambda$$

when $\alpha \to 0$

MB = Moxwell Bultzmann

BE = Bose Einstein

FD = Fermi-Dirac

$$\boxed{u(\lambda)d\lambda = \frac{8\pi hc}{\lambda^5}\frac{1}{e^{hc/\lambda KT}-1}d\lambda}$$

This is planck's law of radiation in terms of wavelength.

⑭

Q Comparison between MB statistics, BE statistics and FD statistics.

	MB	BE	FD
i)	Particles are identical and distinguishable	(i) Particles are identical and indistinguishable	i) Particles are identical and indistinguishable
ii)	Phase space cell have any Volume	ii) Volume of phase space cell is equal to $h_0^3 = h^3$ where h is planck's Constant	ii) Volume of phase space cell is equal to $h_0^3 = h^3$ where h is planck Constant
iii)	$\eta(u)du = \frac{g(u)du}{e^{\alpha}e^{u/KT}}$	iii) $\eta(u)du = \frac{g(u)du}{e^{\alpha}e^{u/KT}-1}$	iii) $\eta(u)du = \frac{g(u)du}{e^{\alpha}e^{u/KT}+1}$
iv)	Phase space cell Can be divided into large number of cell	iv) Phase space cell can be divided into limited cell	iv) Phase space cells can be divided into limited cell
v)	A macro state can have more than one micro state	v) A macro states have only one micro states	v) A macro states have only one micro state

assembly (समष्टि) T_f = Fermi temp

vi) MB statistics is classical statistics	vi) BA statistics is Quantum statics	vi) FD statistics is Quantum statistics
vii) Planck's law of radiation can not be deduce	vii) Planck's law of radiation can be deduce	vii) Planck's law of radiation can be deduce.

Q What is degeneracy in Fermi dirac gas.

Ans An assembly of fermion (electron) is called Fermi - Dirac gas and obeys Pauli exclusion principle. Fermi - Dirac distribution law is given by

$$n(u)du = \frac{g(u)du}{e^{\alpha} e^{u/KT} + 1}$$

where $A = \frac{1}{e^{\alpha}}$

When $T > T_f$

i.e At intermidiate temperature the fermi gas is slightly degenerated and it is called weak deneracy. In this case fermi energy u_f is negative and α is positive then A is less than 1.

When α is large and negative then $A >> 1$.

Condensation Gas to liquid

As degeneration increases with increase of A therefore in this case degeneracy becomes more important and it is called strong degeneracy.

Q. What do you mean by Bose Condensation?

Ans The particle obeying Bose Einstein statistics are called Boson which are identical and indistinguishable particle. When the temperature of B-E gas decreases below the critical temperature, the number of particles in the ground state rapidly increases. The rapid increase in the number of particles of the ground state below critical temperature of Bose Einstein gas is called BE condensation.

In case of Helium gas in liquid state the critical temperature is 3.12K therefore degeneration and condensation of Helium must start as the experimental value comes 2.19 K which is close to the calculated value.

Below this temperature the liquid Helium gives remarkable property of a super fluid i.e 2.9 K is essential temperature for B.E condensation.

8-14

Q. Deduce Wein's Displacement law from planck's law of radiation

Ans Planck's law of radiation in terms of wavelength is given by

$$E_\lambda d\lambda = \frac{8\pi hc}{\lambda^5} \frac{d\lambda}{e^{hc/\lambda KT}-1}$$

$$E_\lambda = \frac{8\pi hc}{\lambda^5} \frac{1}{e^{hc/\lambda KT}-1}$$

Diff. w.r.t. λ we get

$$\frac{dE_\lambda}{d\lambda} = 0$$

or $$\frac{d}{d\lambda}\left[\frac{8\pi hc}{\lambda^5} \frac{1}{e^{hc/\lambda KT}-1}\right] = 0$$

or $$8\pi hc \frac{d}{d\lambda}\left[\lambda^{-5} \cdot \left(e^{hc/\lambda KT}-1\right)^{-1}\right] = 0$$

or $$\left[\left(e^{hc/\lambda KT}-1\right)^{-1} \cdot (-5)\lambda^{-6} + \lambda^{-5} \cdot (-1)\left(e^{hc/\lambda KT}-1\right)^{-2} \cdot \left(e^{hc/\lambda KT} \cdot \frac{hc}{KT} \cdot \left(\frac{-1}{\lambda^2}\right) - 0\right)\right] = \frac{0}{8\pi hc}$$

or $$\left[\frac{-5}{\lambda^6\left(e^{hc/\lambda KT}-1\right)} + \frac{hc}{KT} \frac{1}{\lambda^7} \frac{1 \cdot e^{hc/\lambda KT}}{\left(e^{hc/\lambda KT}-1\right)^2}\right] = 0$$

or $$\frac{1}{\lambda^6}\left[\frac{-5}{e^{hc/\lambda KT}-1} + \frac{hc}{\lambda KT} \frac{e^{hc/\lambda KT}}{\left(e^{hc/\lambda KT}-1\right)^2}\right] = 0$$

or $$\frac{1}{\lambda^6} \times \frac{1}{e^{hc/\lambda KT}-1}\left[-5 + \frac{hc}{\lambda KT} \frac{e^{hc/\lambda KT}}{e^{hc/\lambda KT}-1}\right] = 0$$

physical significance

This law is used to find the colour of star and also gives the explanation of Green house effect

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$$\text{or} \quad -5 + \frac{hc}{\lambda KT} \frac{e^{hc/\lambda KT}}{e^{hc/\lambda KT} - 1} = 0$$

putting $\frac{hc}{\lambda KT} = x$

$$-5 + x \frac{e^{x}}{e^{x} - 1} = 0$$

$$\Rightarrow \frac{x e^{x}}{e^{x} - 1} = 5$$

The only solution of this eqn is, $x = 4.965$

$$\therefore \frac{hc}{\lambda KT} = 4.965$$

$$\Rightarrow \frac{hc}{4.965 K} = \lambda T$$

$$\Rightarrow \lambda T = \frac{hc}{4.965 K}$$

$$\boxed{\therefore \lambda T = \text{Constant}}$$

This is weins Displacement Low

PP-14

Q ~~~~ Derive Bose Einstein Distribution Low

Ans Let us Consider an isolated system of non interacting identical particle (boson) whose spin angular momentum are zero or integral multiple of $\hbar$. The system is ~~a~~ in Equilibrium at absolute temperature T with total energy u, Volume v and total number of Particles n

The statistical theory is based on the basic postulates of statistical mechanics with the additional assumption which are as follows

i) The particles are indistinguishable and any number of particles can occupy a single cell in phase space (or any quantum state)

ii) The size of the cell cannot be less than h^3
where $h = 6.6 \times 10^{-34}$ Js = planck's Constant

iii) The number of phase space cell is comparable with the number of particles.
occupation index $\frac{n_i}{g_i} \approx 1$

The number of different arrangement for ith level having n_i distinguishable particle to be distributed among its g_i cells is given by

$$\omega_i = g_i \frac{(n_i + g_i - 1)!}{n_i!\, g_i!} = \frac{(n_i + g_i - 1)!}{n_i!\,(g_i - 1)!} \qquad -1$$

Therefore total number of different arrangement for all the n particles of the system gives thermodynamic probability

$$\omega(n_1, n_2, n_3, \dots n_K) = \frac{(n_1 + g_1 - 1)!}{n_1!\,(g_1 - 1)!} \times \frac{(n_2 + g_2 - 1)!}{n_2!\,(g_2 - 1)!} \times \dots \frac{(n_K + g_K - 1)!}{n_K!\,(g_K - 1)!}$$

$$\omega = \sum_{i=1}^{K} \frac{(n_i+g_i-1)!}{n_i!\,(g_i-1)!} \quad -2.$$

Since n_i and g_i are very large Quantities hence we can neglected 1 from above Equation.

$$\therefore \omega = \sum_{i=1}^{K} \frac{(n_i+g_i)!}{n_i!\,g_i!} \quad -3.$$

The Condition for maximum value of $\log\omega$ is

$$d[\log\omega] = 0 \quad -4$$

Since ω is function of $n_1, n_2, n_3, \dots n_K$

$$\therefore d\log\omega = \frac{\partial \log\omega}{\partial n_1} dn_1 + \frac{\partial \log\omega}{\partial n_2} dn_2 + \dots + \frac{\partial \log\omega}{\partial n_K} dn_K$$

$$d\log\omega = \sum_{i=1}^{K} \frac{\partial \log\omega}{\partial n_i} dn_i \quad -5.$$

Hence Equation 4 becomes

$$\sum_{i=1}^{K} \frac{\partial \log\omega}{\partial n_i} dn_i = 0 \quad -6$$

The Condition for most probable distribution is given by

$$\frac{\partial \log\omega}{\partial n_i} - \alpha - \beta u_i = 0 \quad 7.$$

Taking log on both side of Eqn 3. we get

$$\log\omega = \sum_{i=1}^{K} \left[\log(n_i+g_i)! - \log n_i! - \log g_i! \right] \quad -8$$

From stering Formula

$$\log(n_i+g_i)! = (n_i+g_i)\log(n_i+g_i) - (n_i+g_i)$$

$$\Rightarrow \quad \log n_i! = n_i \log n_i - n_i$$

$$\log g_i! = g_i \log g_i - g_i$$

Putting these value in Equation 8

$$\log\omega = \sum_{i=1}^{k} \Big[(n_i+g_i)\log(n_i+g_i) \cancel{-n_i} \cancel{-g_i} - n_i \log n_i \cancel{+n_i} - g_i \log g_i \cancel{+g_i} \Big]$$

$$\text{or} \quad \log\omega = \sum_{i=1}^{k} \Big[(n_i+g_i)\log(n_i+g_i) - n_i \log n_i - g_i \log g_i \Big] \quad -9$$

Diff w.r.t n_i taking g_i fixed

$$\frac{\partial \log\omega}{\partial n_i} = \Big[\log(n_i+g_i)(1+0) + \cancel{(n_i+g_i)} \frac{1}{\cancel{(n_i+g_i)}} - \log n_i \times 1 - \cancel{n_i} \times \frac{1}{\cancel{n_i}} - 0 \Big]$$

$$\text{or} \quad \frac{\partial \log\omega}{\partial n_i} = \Big[\log(n_i+g_i) \cancel{+1} - \log n_i \cancel{-1} \Big]$$

$$\text{or} \quad \frac{\partial \log\omega}{\partial n_i} = \log \frac{n_i+g_i}{n_i} \quad -10$$

Putting this value in Equation 7

$$\log \frac{n_i+g_i}{n_i} - \alpha - \beta u_i = 0$$

$$\log \frac{n_i + g_i}{n_i} = \alpha + \beta u_i$$

or $$\frac{n_i + g_i}{n_i} = e^{\alpha + \beta u_i}$$

$$\Rightarrow 1 + \frac{g_i}{n_i} = e^{\alpha + \beta u_i}$$

$$\Rightarrow \frac{g_i}{n_i} = e^{\alpha + \beta u_i} - 1$$

$$\Rightarrow \frac{n_i}{g_i} = \frac{1}{e^{\alpha + \beta u_i} - 1}$$

$$\boxed{n_i = \frac{g_i}{e^{\alpha + \beta u_i} - 1}} \quad -11$$

Equation 11 is known as Bose Einstein distribution Low

If $g(u)du$ is the number of energy state and $n(u)du$ is the number of particle between the energy range u and $u+du$ then

$$g_i = g(u)du$$

$$n_i = n(u)du$$

$$\boxed{\therefore \; n(u)du = \frac{g(u)du}{e^{\alpha + u_i/KT} - 1}}$$

where $\beta = \frac{1}{KT}$